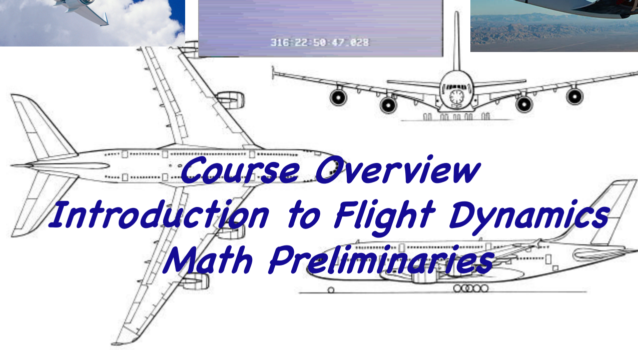
Aircraft Flight Dynamics

Robert Stengel
MAE 331, Princeton University, 2018





Course Overview
Introduction to Flight Dynamics
Math Preliminaries

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At the End of the Course, you should be able to:

- <u>Understand</u> aircraft configuration aerodynamics, performance, stability, and control
- <u>Estimate</u> an aircraft's aerodynamic characteristics from geometric and inertial properties
- <u>Analyze</u> linear and nonlinear dynamic systems
- <u>Recognize</u> airplane modes of motion and their significance
- <u>Compute</u> aircraft motions
- <u>Appreciate</u> historical development of aviation

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Syllabus, First Half

- Introduction, Math Preliminaries
- Point Mass Dynamics
- Aerodynamics of Airplane Configurations
  - Forces & Moments
  - 2-D & 3-D
  - Low- & High-Speed
- Cruising Flight Performance
  - Power & Thrust
  - Flight Envelope
- Gliding, Climbing, and Turning Performance
- Nonlinear, 6-DOF Equations of Motion
- Aircraft Control Devices and Systems

Details, reading, homework assignments, and references at
http://blackboard.princeton.edu/

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Syllabus, Second Half

- Linearized Equations of Motion
- Longitudinal Dynamics
- Lateral-Directional Dynamics
- Analysis of Linear Systems
  - Time Response
  - Transfer Functions and Frequency Response
  - Root Locus Analysis
- Flying Qualities Criteria
- Maneuvering at High Angles and Rates
- Aeroelasticity and Fuel Slosh
- Special Problems

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You're interested in MAE 331 because ...?

Details

- *Lecture: 3-4:20, J-201, Tue & Thu, E-Quad*
- *Precept: 7:30-8:20, J-201, Mon*
- *Engineering, science, & math*
- *Case studies, historical context*
- *~8 homework assignments*
- *Office hours: 1:30-2:30, MW, D-202, or any time my door is open. e-mail ahead, if possible*
- *Assistant in Instruction: Office hours: TBD*

Details

- **Lecture slides**
  - pdfs from all 2016 lectures are available now at http://www.stengel.mycpanel.princeton.edu/MAE331.html
  - pdf for current (2018) lecture on Blackboard morning of class or day before

- **GRADING**
  - Assignments: 50%
  - Term Paper: 30%
  - Class participation: 10%
  - Quick Quiz (10, 5 min): 10%

See: http://cte.virginia.edu/resources/grading-class-participation-2/

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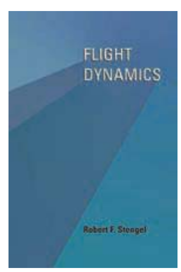
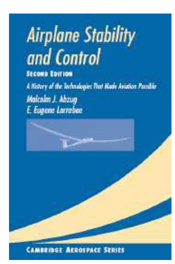
Text and References

- Science, Engineering, and Math:
  - *Flight Dynamics*, RFS, Princeton University Press, 2004
- Case Studies, Historical Context
  - *Airplane Stability and Control*, Abzug and Larrabee, Cambridge University Press, 2002
- Technical Report PDFs on Blackboard
- Virtual reference book





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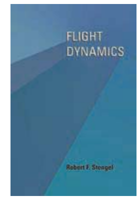
Flight Dynamics Book and Computer Code



- Programs accessible from the *Flight Dynamics* web page
  - *http://www.princeton.edu/~stengel/FlightDynamics.html*
- ... or directly
- *ERRATA* for the book are listed there
- 6-degree-of-freedom nonlinear simulation of a business jet aircraft (MATLAB)
  - *http://www.princeton.edu/~stengel/FDcodeB.html*
- Linear system analysis (MATLAB)
  - *http://www.princeton.edu/~stengel/FDcodeC.html*
- Paper airplane simulation (MATLAB)
  - *http://www.princeton.edu/~stengel/PaperPlane.html*
- Performance analysis of a business jet aircraft (Excel)
  - *http://www.princeton.edu/~stengel/Example261.xls*

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Quick Quizzes *First 5 Minutes of 10 Classes*

- *One question about the lectures and reading assignments from the previous week*
- *Largely qualitative but may require simple calculations*
- *Be sure to bring a pencil, paper, and calculator to class*

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Homework Assignments

- *Groups of 2 or 3 students for all assignments*
- *Team members for each assignment will be*
  - *different*
  - *chosen using a spreadsheet and random number generator (TBD)*
- *Each member of each team will receive the same grade as the others*
- *Identify who did what on each assignment*
- *Submit via* http://blackboard.princeton.edu/

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Flight Tests Using Balsa Glider and Cockpit Flight Simulator

- *Compare actual flight of the glider with trajectory simulation*

- *In your Cessna 172:*
- *Takeoff from Princeton Airport*
- *Fly over Carnegie Lake*
- *Land at Princeton Airport*

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Assignment #1

- Document the physical characteristics and flight behavior of a balsa glider
  - Everything that you know about the physical characteristics
  - Everything that you know about the flight characteristics
- 2- or 3-person team, joint write-up
- Team assignments on http://blackboard.princeton.edu/

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Luke Nash's Biplane Glider Flight #1 (MAE 331, 2008)

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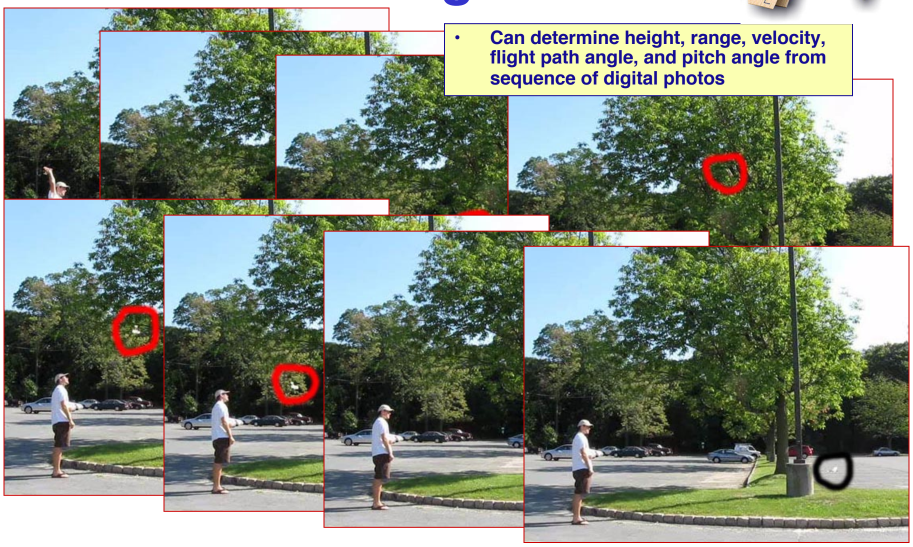
Frame Grab Sequence of Luke Nash's Flight

- Can determine height, range, velocity, flight path angle, and pitch angle from sequence of digital photos



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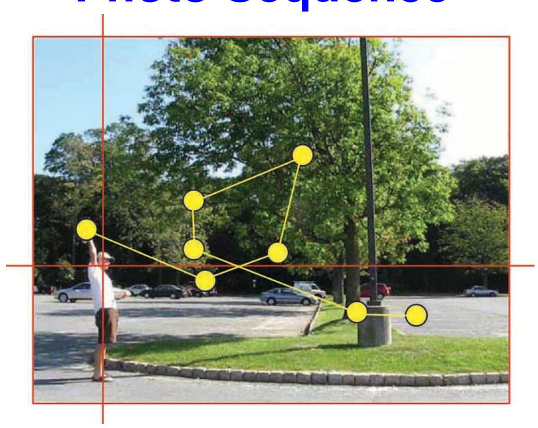
Trajectory Estimation from Photo Sequence



Smaller, fixed-interval time steps
Interpolation and differencing

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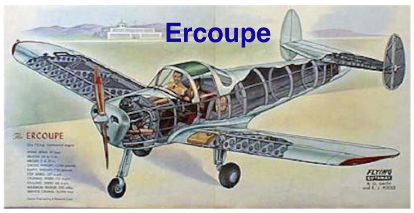
Stability and Control Case Studies



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Reading Assignments

- *Do Flight Dynamics reading before class*
- *Case Studies/Historical Context:*

  Airplane Stability and Control
  - *10-minute synopses by groups of 3 students*
    - *Principal subject/scope of chapter*
    - *Technical ideas needed to understand chapter*
    - *When did the events occur?*
    - *3 main "takeaway points" or conclusions*
    - *3 most surprising or remarkable facts*

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Goals for Airplane Design

- **Shape of airplane determined by purpose**
- **Safety, handling, performance, functioning, and comfort**
- **Agility vs. sedateness**
- **Control surfaces adequate to produce needed moments (i.e., torques)**
- **<u>Tradeoffs</u>, *e.g.*, center of mass location**
  - too far forward increases unpowered control-stick forces
  - too far aft degrades static stability

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Configuration Driven By The Mission and Flight Envelope

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Inhabited Air Vehicles

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Uninhabited Air Vehicles (UAV)

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Introduction to Flight Dynamics

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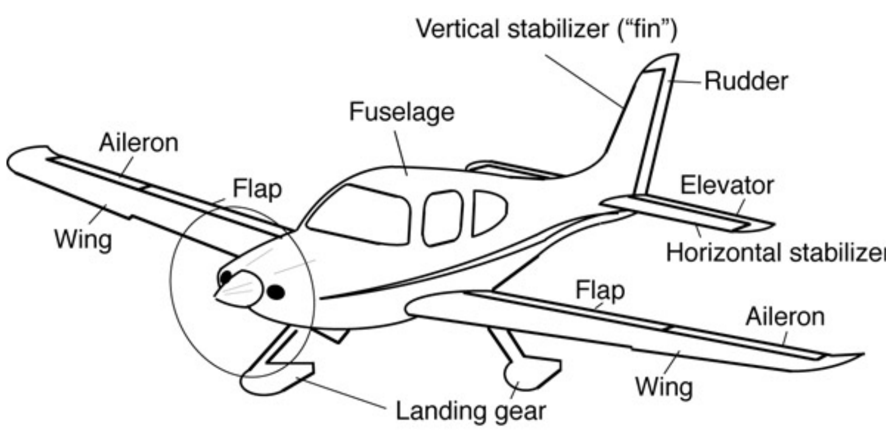
Airplane Components



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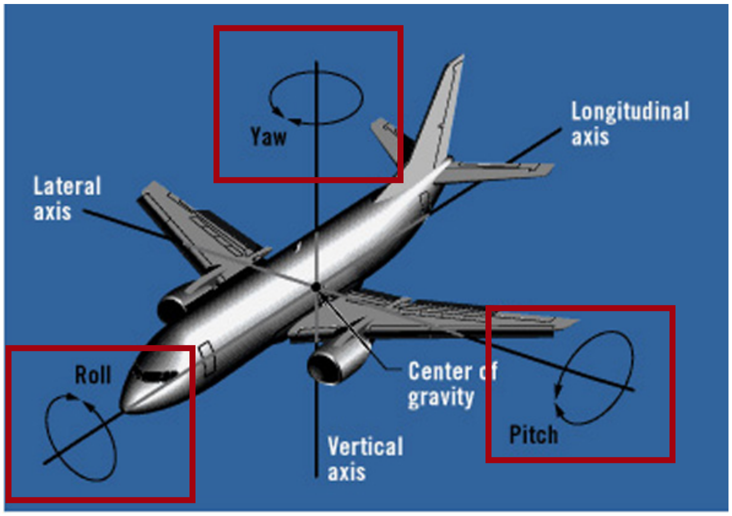
Airplane Rotational Degrees of Freedom



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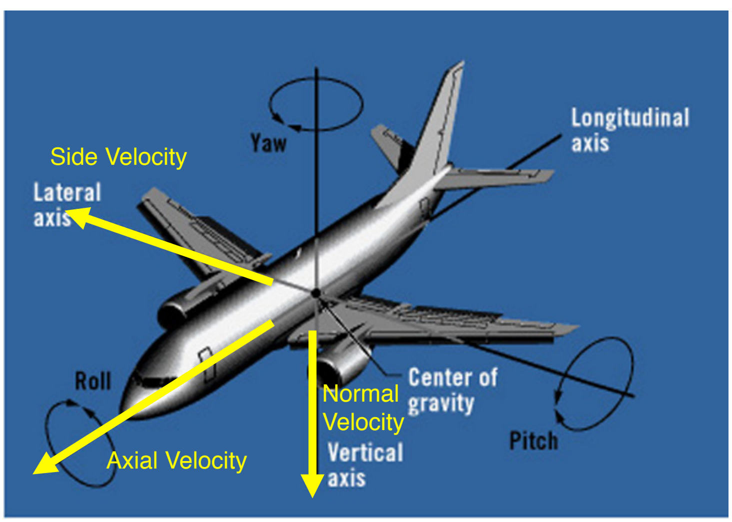
Airplane Translational Degrees of Freedom



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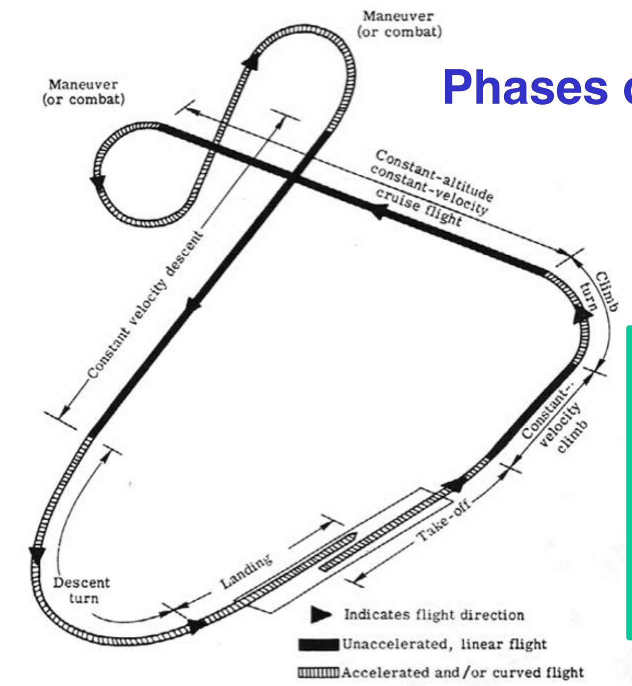
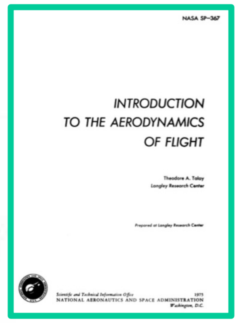
Phases of Flight





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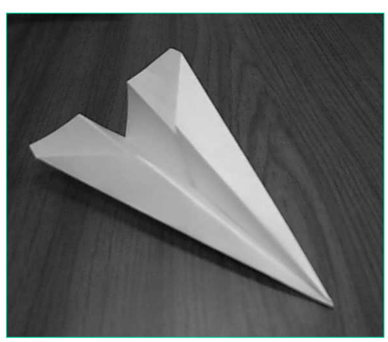
Flight of a Paper Airplane



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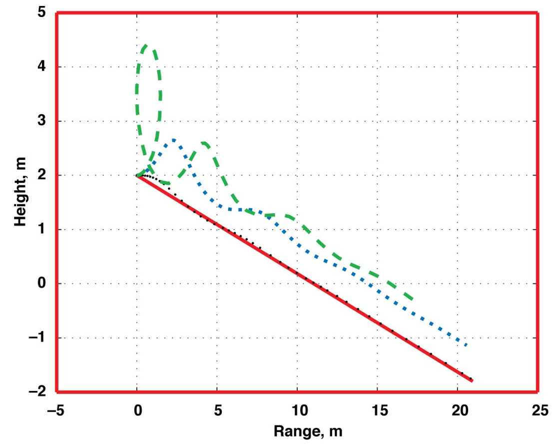
Flight of a Paper Airplane
Example 1.3-1, *Flight Dynamics*

- Equations of motion integrated numerically to estimate the flight path
- Red: Equilibrium flight path
- Black: Initial flight path angle = 0
- Blue: plus increased initial airspeed
- Green: loop



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Flight of a Paper Airplane
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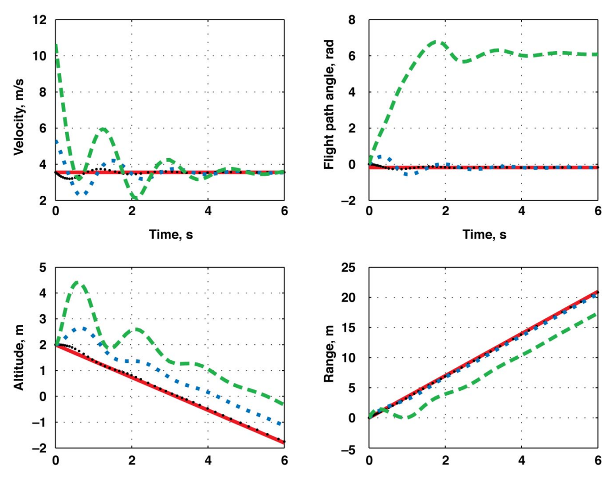

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Assignment #2

- *Compute the trajectory of a balsa glider*
- *Simulate using equations of motion*
- *Compare to the actual flight of the glider (HW #1)*
- *Similar to the flight of a paper airplane*

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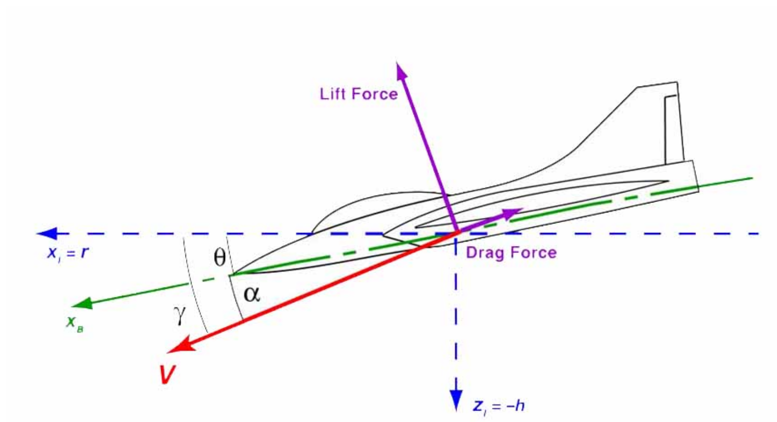
Gliding Flight



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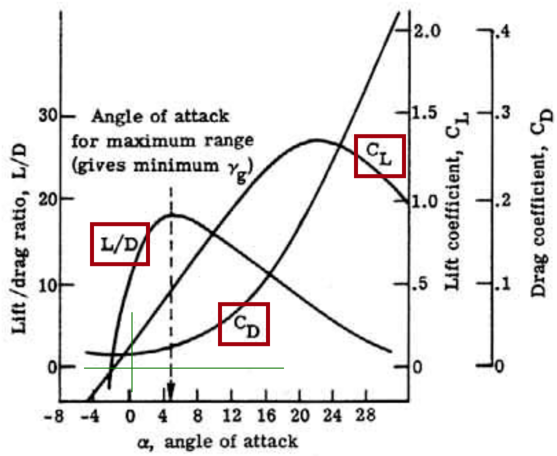
Configuration Aerodynamics



(b) Glide aerodynamic characteristics.

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Math Preliminaries

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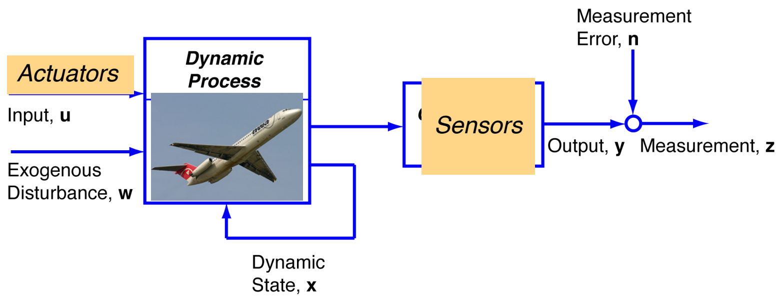
Dynamic Systems



Dynamic Process: Current state depends on prior state

x	= dynamic state
u	= input
w	= exogenous disturbance
p	= parameter
t or *k*	= time or event index

Observation Process: Measurement may contain error or be incomplete

y	= output (error-free)
z	= measurement
n	= measurement error

All of these quantities are multidimensional
They can be expressed as vectors

Notation for Scalars and Vectors

- **Scalar: usually lower case:** ***a, b, c, …, x, y, z***

$$a = 12; \quad b = 7; \quad c = a + b = 19; \quad x = a + b^2 = 12 + 49 = 61$$

- **Vector**: usually bold or with underbar: **x** or <u>x</u>
  - Ordered set
  - Column of scalars
  - Dimension = *n* x 1

$$\mathbf{a} = \begin{bmatrix} 2 \\ -7 \\ 16 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Matrices and Transpose

- **Matrix**: usually bold capital or capital: **F** or F
  - Dimension = (*m* x *n*)

$$\mathbf{x} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}; \quad \mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \\ l & m & n \end{bmatrix}$$

(3×1) (4×3)

- **Transpose**: interchange rows and columns

$$\mathbf{x}^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

$$\mathbf{A}^T = \begin{bmatrix} a & d & g & l \\ b & e & h & m \\ c & f & k & n \end{bmatrix}$$

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Multiplication

- **Operands must be conformable**
- **Multiplication of vector by scalar is associative, commutative, and distributive**

$$a\mathbf{x} = \mathbf{x}a = \begin{bmatrix} ax_1 \\ ax_2 \\ ax_3 \end{bmatrix}$$

$$a(\mathbf{x}+\mathbf{y}) = (\mathbf{x}+\mathbf{y})a = (a\mathbf{x}+a\mathbf{y})$$

$\dim(\mathbf{x}) = \dim(\mathbf{y})$

$$a\mathbf{x}^T = \begin{bmatrix} ax_1 & ax_2 & ax_3 \end{bmatrix}$$

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Addition

- Conformable vectors and matrices are added term by term

$$\mathbf{x} = \begin{bmatrix} a \\ b \end{bmatrix} \quad ; \quad \mathbf{z} = \begin{bmatrix} c \\ d \end{bmatrix}$$

$$\mathbf{x} + \mathbf{z} = \begin{bmatrix} a + c \\ b + d \end{bmatrix}$$

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Inner Product

- Inner (dot) product of vectors produces scalar result

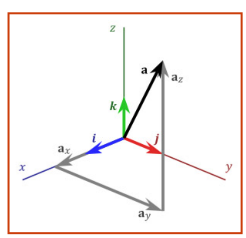
$$\mathbf{x}^T\mathbf{x} = \mathbf{x} \bullet \mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$(1 \times m)(m \times 1) = (1 \times 1)$

$$= (x_1^2 + x_2^2 + x_3^2)$$

- Length (or magnitude) of vector is square root of dot product

$$= (x_1^2 + x_2^2 + x_3^2)^{1/2}$$



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Vector Transformation

- **Matrix-vector product transforms one vector into another**
- **Matrix-matrix product produces a new matrix**

$$\mathbf{y} = \mathbf{A}\mathbf{x} = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -5 & 7 \\ 4 & 1 & 8 \\ -9 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$(n \times 1) = (n \times m)(m \times 1)$$

$$= \begin{bmatrix} (2x_1 + 4x_2 + 6x_3) \\ (3x_1 - 5x_2 + 7x_3) \\ (4x_1 + x_2 + 8x_3) \\ (-9x_1 - 6x_2 - 3x_3) \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

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Derivatives and Integrals of Vectors

Derivatives and integrals of vectors are vectors of derivatives and integrals

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} dx_1/dt \\ dx_2/dt \\ dx_3/dt \end{bmatrix}$$

$$\int \mathbf{x}\, dt = \begin{bmatrix} \int x_1\, dt \\ \int x_2\, dt \\ \int x_3\, dt \end{bmatrix}$$

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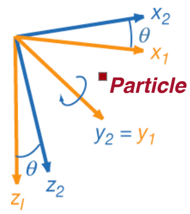

Matrix Inverse

Transformation

$$\mathbf{x}_2 = \mathbf{A}\mathbf{x}_1$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_1$$

Inverse Transformation

$$\mathbf{x}_1 = \mathbf{A}^{-1}\mathbf{x}_2$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2$$

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Matrix Identity and Inverse

- **Identity matrix**: **no change** when it multiplies a conformable vector or matrix

$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{y} = \mathbf{I}\mathbf{y}$$

- **A non-singular square matrix multiplied by its inverse forms an *identity matrix***

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

$$\mathbf{A}\mathbf{A}^{-1} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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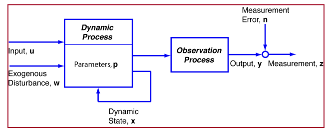

Mathematical Models of Dynamic Systems are Differential Equations

Continuous-time dynamic process:
Vector Ordinary Differential Equation

$$\dot{\mathbf{x}}(t) \triangleq \frac{d\mathbf{x}(t)}{dt} = \mathbf{f}[\mathbf{x}(t),\mathbf{u}(t),\mathbf{w}(t),\mathbf{p}(t),t]$$

$$\dim(\mathbf{x}) = (n \times 1)$$
$$\dim(\mathbf{f}) = (n \times 1)$$
$$\dim(\mathbf{u}) = (m \times 1)$$
$$\dim(\mathbf{w}) = (s \times 1)$$
$$\dim(\mathbf{p}) = (l \times 1)$$

Output Transformation

$$\mathbf{y}(t) = \mathbf{h}[\mathbf{x}(t),\mathbf{u}(t)]$$

$$\dim(\mathbf{y}) = (r \times 1)$$
$$\dim(\mathbf{h}) = (r \times 1)$$

Measurement with Error

$$\mathbf{z}(t) = \mathbf{y}(t) + \mathbf{n}(t)$$

$$\dim(\mathbf{z}) = (r \times 1)$$
$$\dim(\mathbf{n}) = (r \times 1)$$

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Next Time:

Point-Mass Dynamics
Aerodynamic/Thrust Forces

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Supplemental Material

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Helpful Resources

- Web pages
  - http://blackboard.princeton.edu/
  - http://www.stengel.mycpanel.princeton.edu/MAE331.html
    http://www.stengel.mycpanel.princeton.edu/FlightDynamics.html
- Princeton University Engineering Library (paper and on-line)
  - http://lib-terminal.princeton.edu/ejournals/by_title_zd.asp
  - http://sfx.princeton.edu:9003/sfx_pul/az
- NACA/NASA pubs
  - http://ntrs.nasa.gov/search.jsp

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MAE 331 Course Learning Objectives

(Accreditation Board for Engineering and Technology)

Course Learning Objectives	ABET Criterion 3
Understanding of the dynamics and control of aircraft.	a
Ability to estimate aerodynamic coefficients and stability derivatives from aircraft geometry and flight envelope.	a, c
Facility in analyzing mathematical descriptions of the rigid-body motions of flying vehicles.	a
Ability to estimate the performance, stability, and control characteristics of aircraft.	b
Development of appreciation for flight-testing methods and results.	b, k
Ability to apply systems-engineering approach to the analysis, design, and testing of aircraft.	b, c
Demonstration of ability to work in multidisciplinary teams.	d
Demonstration of computational problem-solving, through thorough knowledge, application, and development of analytical software.	e, k
Appreciation of the historical context within which airplanes have evolved to present-day configurations.	f, h, i, j
Competence in presenting ideas.	g