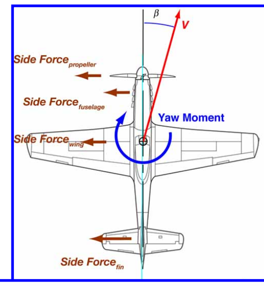
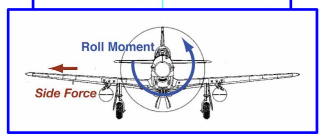
Linearized Lateral-Directional Equations of Motion

Robert Stengel, Aircraft Flight Dynamics MAE 331, 2018

Learning Objectives

- 6th-order -> 4th-order -> hybrid equations
- Dynamic stability derivatives
- Dutch roll mode
- Roll and spiral modes





Reading:
Flight Dynamics
574-591

http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

1

6-Component Lateral-Directional Equations of Motion

$$\dot{v} = Y_B / m + g\sin\phi\cos\theta - ru + pw$$

$$\dot{y}_I = (\cos\theta\sin\psi)u + (\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi)v + (-\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi)w$$

$$\dot{p} = \left(I_{zz}L_B + I_{xz}N_B - \left\{I_{xz}\left(I_{yy} - I_{xx} - I_{zz}\right)p + \left[I_{xz}^2 + I_{zz}\left(I_{zz} - I_{yy}\right)\right]r\right\}q\right) \div \left(I_{xx}I_{zz} - I_{xz}^2\right)$$

$$\dot{r} = \left(I_{xz}L_B + I_{xx}N_B - \left\{I_{xz}\left(I_{yy} - I_{xx} - I_{zz}\right)r + \left[I_{xz}^2 + I_{xx}\left(I_{xx} - I_{yy}\right)\right]p\right\}q\right) \div \left(I_{xx}I_{zz} - I_{xz}^2\right)$$

$$\dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta$$

$$\dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \mathbf{x}_{LD_6}$$

$$\begin{bmatrix} v \\ y \\ p \\ r \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} \text{Side Velocity} \\ \text{Crossrange} \\ \text{Body-Axis Roll Rate} \\ \text{Body-Axis Yaw Rate} \\ \text{Roll Angle about Body } x \text{ Axis} \\ \text{Yaw Angle about Inertial } x \text{ Axis} \end{bmatrix}$$

2

4- Component Lateral-Directional Equations of Motion

Nonlinear Dynamic Equations, neglecting crossrange and yaw angle

$$\dot{v} = Y_B / m + g\sin\phi\cos\theta - ru + pw$$

$$\dot{p} = \left(I_{zz}L_B + I_{xz}N_B - \left\{I_{xz}\left(I_{yy} - I_{xx} - I_{zz}\right)p + \left[I_{xz}^2 + I_{zz}\left(I_{zz} - I_{yy}\right)\right]r\right\}q\right) \div \left(I_{xx}I_{zz} - I_{xz}^2\right)$$

$$\dot{r} = \left(I_{xz}L_B + I_{xx}N_B - \left\{I_{xz}\left(I_{yy} - I_{xx} - I_{zz}\right)r + \left[I_{xz}^2 + I_{xx}\left(I_{xx} - I_{yy}\right)\right]p\right\}q\right) \div \left(I_{xx}I_{zz} - I_{xz}^2\right)$$

$$\dot{\phi} = p + \left(q\sin\phi + r\cos\phi\right)\tan\theta$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \mathbf{x}_{LD_4}$$

$$\begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} = \begin{bmatrix} \text{Side Velocity} \\ \text{Body-Axis Roll Rate} \\ \text{Body-Axis Yaw Rate} \\ \text{Roll Angle about Body } x \text{ Axis} \end{bmatrix}$$

3

Lateral-Directional Equations of Motion

Assuming Steady, Level Longitudinal Flight

Longitudinal variables are constant

$$\dot{v} = Y_B / m + g\sin\phi\cos\theta_N - ru_N + pw_N$$

$$\dot{p} = \left(I_{zz}L_B + I_{xz}N_B\right) / \left(I_{xx}I_{zz} - I_{xz}^2\right)$$

$$\dot{r} = \left(I_{xz}L_B + I_{xx}N_B\right) / \left(I_{xx}I_{zz} - I_{xz}^2\right)$$

$$\dot{\phi} = p + \left(r\cos\phi\right)\tan\theta_N$$

$$q_N = 0$$
$$\gamma_N = 0$$
$$\theta_N = \alpha_N$$

4

Lateral-Directional Force and Moments in Steady, Level Flight

Dynamic pressure is constant

$$Y_B = C_{Y_B} \frac{1}{2} \rho_N V_N^2 S$$

$$L_B = C_{l_B} \frac{1}{2} \rho_N V_N^2 Sb$$

$$N_B = C_{n_B} \frac{1}{2} \rho_N V_N^2 Sb$$

Body-Axis Side Force
Body-Axis Rolling Moment
Body-Axis Yawing Moment

5

Linearized Lateral-Directional Equations of Motion in Steady, Level Flight

6

Body-Axis Perturbation Equations of Motion

$$\begin{bmatrix} \Delta\dot{v}(t) \\ \Delta\dot{p}(t) \\ \Delta\dot{r}(t) \\ \Delta\dot{\phi}(t) \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & F_{43} & F_{44} \end{bmatrix} \begin{bmatrix} \Delta v(t) \\ \Delta p(t) \\ \Delta r(t) \\ \Delta\phi(t) \end{bmatrix} + \left[Control \right] + \left[Disturbance \right]$$

7

Body-Axis Perturbation Variables

$$\begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} = \begin{bmatrix} \text{Side Velocity Perturbation} \\ \text{Body-Axis Roll Rate Perturbation} \\ \text{Body-Axis Yaw Rate Perturbation} \\ \text{Roll Angle about Body } x \text{ Axis Perturbation} \end{bmatrix}$$

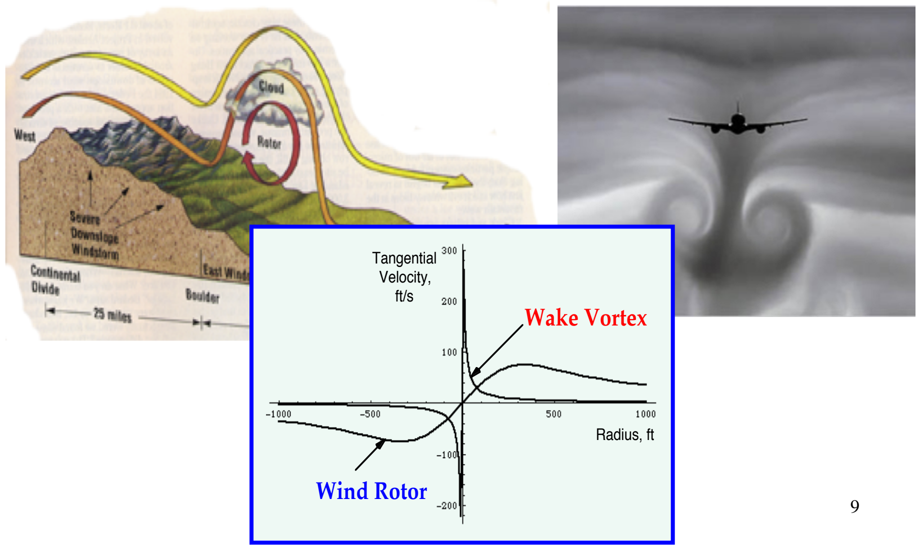
$$\begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix} = \begin{bmatrix} \Delta\delta A \\ \Delta\delta R \end{bmatrix} = \begin{bmatrix} \text{Aileron Perturbation} \\ \text{Rudder Perturbation} \end{bmatrix}$$

$$\begin{bmatrix} \Delta w_1 \\ \Delta w_2 \end{bmatrix} = \begin{bmatrix} \Delta v_{wind} \\ \Delta p_{wind} \end{bmatrix} = \begin{bmatrix} \text{Side Wind Perturbation} \\ \text{Vortical Wind Perturbation} \end{bmatrix}$$

8

Vortical Wind Perturbations

Wind Rotor ***Wake Vortex***



9

Dimensional Stability Derivatives

Stability Matrix

$$\begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & & \end{bmatrix} = \begin{bmatrix} Y_v & \left(Y_p + w_N\right) & \left(Y_r - u_N\right) & g\cos\theta_N \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & \tan\theta_N & 0 \end{bmatrix}$$

10

Dimensional Control- and Disturbance-Effect Derivatives

Control Effect Matrix

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \\ G_{31} & G_{32} \\ G_{41} & G_{42} \end{bmatrix} = \begin{bmatrix} Y_{\delta A} & Y_{\delta R} \\ L_{\delta A} & L_{\delta R} \\ N_{\delta A} & N_{\delta R} \\ 0 & 0 \end{bmatrix}$$

Disturbance Effect Matrix

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \\ L_{31} & L_{32} \\ L_{41} & L_{42} \end{bmatrix} = \begin{bmatrix} Y_{v} & Y_{p} \\ L_{v} & L_{p} \\ N_{v} & N_{p} \\ 0 & 0 \end{bmatrix}$$

11

LTI Body-Axis Perturbation Equations of Motion

Rolling and yawing motions

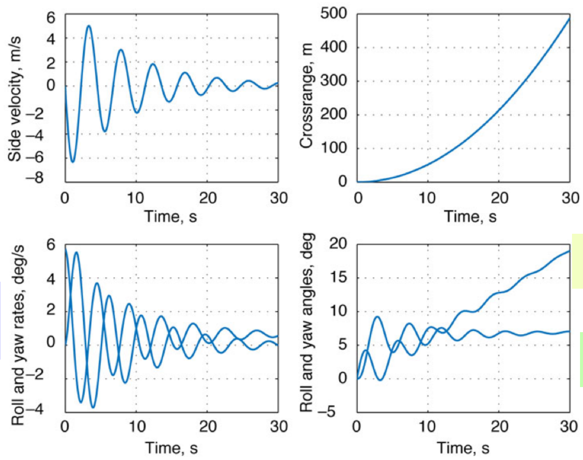
$$\begin{bmatrix} \Delta\dot{v}(t) \\ \Delta\dot{p}(t) \\ \Delta\dot{r}(t) \\ \Delta\dot{\phi}(t) \end{bmatrix} = \begin{bmatrix} Y_v & \left(Y_p + w_N\right) & \left(Y_r - u_N\right) & g\cos\theta_N \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & \tan\theta_N & 0 \end{bmatrix} \begin{bmatrix} \Delta v(t) \\ \Delta p(t) \\ \Delta r(t) \\ \Delta\phi(t) \end{bmatrix}$$
$$+ \begin{bmatrix} Y_{\delta A} & Y_{\delta R} \\ L_{\delta A} & L_{\delta R} \\ N_{\delta A} & N_{\delta R} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta A(t) \\ \Delta\delta R(t) \end{bmatrix} + \begin{bmatrix} Y_v & Y_p \\ L_v & L_p \\ N_v & N_p \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta v_{wind} \\ \Delta p_{wind} \end{bmatrix}$$

12

Linearized Lateral-Directional Response to Initial Yaw Rate

~Dutch-roll response of side velocity

~Spiral response of crossrange



~Dutch-roll response of roll and yaw rates

~Spiral response of yaw angle

~Roll response of roll angle

13

Unusual Aircraft Factoids

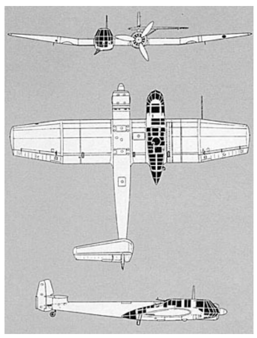
Asymmetrical Aircraft: DC-2-1/2

DC-3 with *DC-2* right wing
Quick fix to fly aircraft out of harm's way during WWII

14

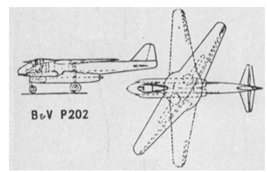
Asymmetric Aircraft - WWII

Blohm und Voss, BV 141



B + V 141 derivatives

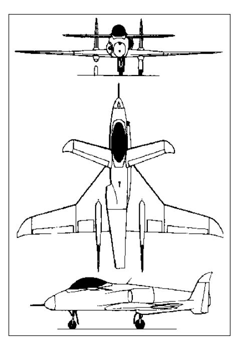
B + V P.202



Recent Asymmetric Aircraft

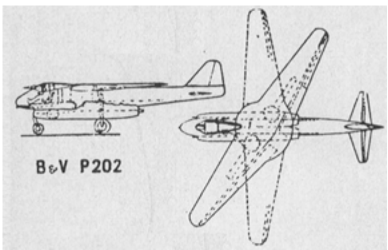
Scaled Composites Boomerang

Scaled Composites Ares



Oblique Wing Concepts

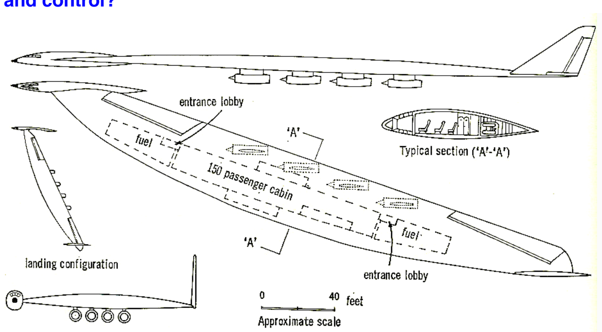
- High-speed benefits of wing sweep without the heavy structure and complex mechanism required for symmetric sweep
- Blohm und Voss, R. T. Jones , Handley-Page concepts
- Improved supersonic L/D by reduction of shock-wave interference and elimination of the fuselage in flying-wing version



17

Handley-Page Oblique Wing Concepts

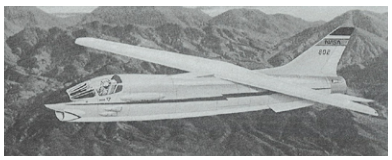
- Advantages
  - 10-20% higher L/D @ supersonic speed (compared to delta planform)
  - Flying wing: no fuselage
- Issues
  - Which way do the passengers face?
  - Where is the cockpit?
  - How are the engines and vertical surfaces swiveled?
  - What does asymmetry do to stability and control?

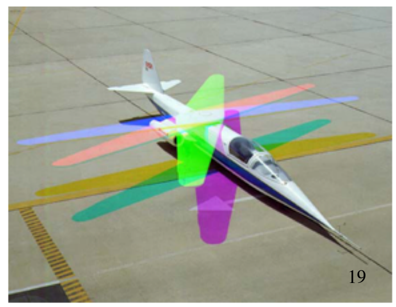


18

NASA Oblique Wing Test Vehicles

- **Stability and control issues abound: The fact that birds and insects are symmetric should give us a clue (though they use huge asymmetry for control)**
  - **Strong aerodynamic and inertial longitudinal-lateral-directional coupling**
  - **High side force at zero sideslip angle**
  - **Torsional divergence of the leading wing**
- **Test vehicles: Various model airplanes, *NASA AD-1*, and *NASA DFBW F-8* (below, not built)**





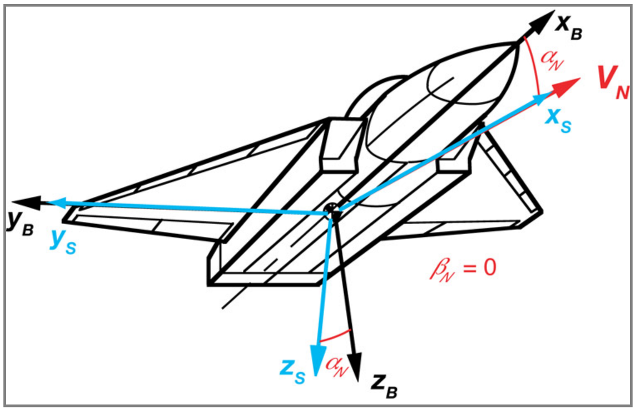
19

Stability Axis Representation of Dynamics

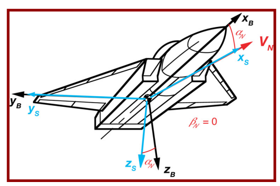
20

Stability Axes

- Stability axes are an alternative set of body axes
- Nominal x axis is offset from the body centerline by the nominal angle of attack, α_N



Transformation from Original Body Axes to Stability Axes



$$\mathbf{H}_B^S = \begin{bmatrix} \cos\alpha_N & 0 & \sin\alpha_N \\ 0 & 1 & 0 \\ -\sin\alpha_N & 0 & \cos\alpha_N \end{bmatrix}$$

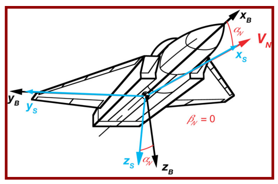
$$\begin{bmatrix} \Delta u \\ \Delta v \\ \Delta w \end{bmatrix}_S = H_B^S \begin{bmatrix} \Delta u \\ \Delta v \\ \Delta w \end{bmatrix}_B \qquad \begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}_S = H_B^S \begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}_B$$

Side velocity (Δv) and pitch rate (Δq) are unchanged by the transformation

Stability-Axis State Vector

Rotate body axes to stability axes

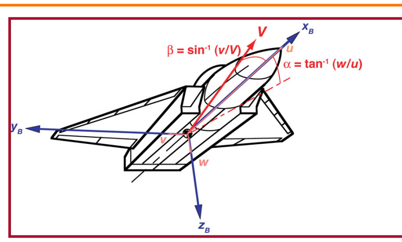
$$\begin{bmatrix} \Delta v(t) \\ \Delta p(t) \\ \Delta r(t) \\ \Delta \phi(t) \end{bmatrix}_{Body-Axis} \Rightarrow \measuredangle \alpha_N \Rightarrow \begin{bmatrix} \Delta v(t) \\ \Delta p(t) \\ \Delta r(t) \\ \Delta \phi(t) \end{bmatrix}_{Stability-Axis}$$



Stability-Axis State Vector

Replace side velocity by sideslip angle

$$\begin{bmatrix} \Delta v(t) \\ \Delta p(t) \\ \Delta r(t) \\ \Delta \phi(t) \end{bmatrix}_{Stability-Axis} \Rightarrow \Delta\beta \approx \frac{\Delta v}{V_N} \Rightarrow \begin{bmatrix} \Delta \beta(t) \\ \Delta p(t) \\ \Delta r(t) \\ \Delta \phi(t) \end{bmatrix}_{Stability-Axis}$$



Stability-Axis State Vector

Revise state order

$$\begin{bmatrix} \Delta\beta(t) \\ \Delta p(t) \\ \Delta r(t) \\ \Delta\phi(t) \end{bmatrix}_{Stability-Axis} \Rightarrow \begin{bmatrix} \Delta r(t) \\ \Delta\beta(t) \\ \Delta p(t) \\ \Delta\phi(t) \end{bmatrix}_{Stability-Axis} = \begin{bmatrix} \text{Stability-Axis Yaw Rate Perturbation} \\ \text{Sideslip Angle Perturbation} \\ \text{Stability-Axis Roll Rate Perturbation} \\ \text{Stability-Axis Roll Angle Perturbation} \end{bmatrix}$$

25

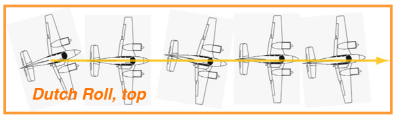
Stability-Axis Lateral-Directional Equations

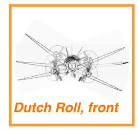
$$\begin{bmatrix} \Delta\dot{r}(t) \\ \Delta\dot{\beta}(t) \\ \Delta\dot{p}(t) \\ \Delta\dot{\phi}(t) \end{bmatrix}_S = \begin{bmatrix} N_r & N_\beta & N_p & 0 \\ \left(\frac{Y_r}{V_N}-1\right) & \frac{Y_\beta}{V_N} & \frac{Y_p}{V_N} & \frac{g}{V_N} \\ L_r & L_\beta & L_p & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_S \begin{bmatrix} \Delta r(t) \\ \Delta\beta(t) \\ \Delta p(t) \\ \Delta\phi(t) \end{bmatrix}_S + \begin{bmatrix} N_{\delta A} & N_{\delta R} \\ \frac{Y_{\delta A}}{V_N} & \frac{Y_{\delta R}}{V_N} \\ L_{\delta A} & L_{\delta R} \\ 0 & 0 \end{bmatrix}_S \begin{bmatrix} \Delta\delta A(t) \\ \Delta\delta R(t) \end{bmatrix} + \begin{bmatrix} N_\beta & N_p \\ \frac{Y_\beta}{V_N} & \frac{Y_p}{V_N} \\ L_\beta & L_p \\ 0 & 0 \end{bmatrix}_S \begin{bmatrix} \Delta\beta_{wind} \\ \Delta p_{wind} \end{bmatrix}$$

26

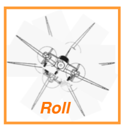
Why Modify the Equations?

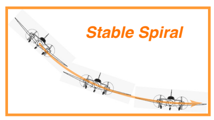
Dutch-roll motion is primarily described by stability-axis yaw rate and sideslip angle





Roll and spiral motions are primarily described by stability-axis roll rate and roll angle





27

Why Modify the Equations?

Effects of **Dutch roll** perturbations on **Dutch roll** motion

Effects of **roll-spira**l perturbations on **Dutch roll** motion

$$\mathbf{F}_{LD} = \left[\begin{array}{c|c} \mathbf{F}_{DR} & \mathbf{F}_{RS}^{DR} \\ \hline \mathbf{F}_{DR}^{RS} & \mathbf{F}_{RS} \end{array}\right] = \left[\begin{array}{c|c} \mathbf{F}_{DR} & small \\ \hline small & \mathbf{F}_{RS} \end{array}\right] \approx \left[\begin{array}{c|c} \mathbf{F}_{DR} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{F}_{RS} \end{array}\right]$$

Effects of **Dutch roll** perturbations on **roll-spiral** motion

Effects of **roll-spiral** perturbations on **roll-spiral** motion

... but are the off-diagonal blocks really small?

28

Stability, Control, and Disturbance Matrices

$$\mathbf{F}_{LD} = \left[\begin{array}{c|c} \mathbf{F}_{DR} & \mathbf{F}_{RS}^{DR} \\ \hline \mathbf{F}_{DR}^{RS} & \mathbf{F}_{RS} \end{array}\right] = \left[\begin{array}{cc|cc} N_r & N_\beta & N_p & 0 \\ \left(\frac{Y_r}{V_N} - 1\right) & \frac{Y_\beta}{V_N} & \frac{Y_p}{V_N} & \frac{g}{V_N} \\ \hline L_r & L_\beta & L_p & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix} = \begin{bmatrix} \Delta r \\ \Delta \beta \\ \Delta p \\ \Delta \phi \end{bmatrix}$$

$$\mathbf{G}_{LD} = \left[\begin{array}{c|c} N_{\delta A} & N_{\delta R} \\ \frac{Y_{\delta A}}{V_N} & \frac{Y_{\delta R}}{V_N} \\ \hline L_{\delta A} & L_{\delta R} \\ 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix} = \begin{bmatrix} \Delta \delta A \\ \Delta \delta R \end{bmatrix}$$

$$\mathbf{L}_{LD} = \left[\begin{array}{c|c} N_\beta & N_p \\ \frac{Y_\beta}{V_N} & \frac{Y_p}{V_N} \\ \hline L_\beta & L_p \\ 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} \Delta w_1 \\ \Delta w_2 \end{bmatrix} = \begin{bmatrix} \Delta \delta A \\ \Delta \delta R \end{bmatrix}$$

29

2nd-Order Approximate Modes of Lateral-Directional Motion

30

2nd-Order Approximations in System Matrices

$$\mathbf{F}_{LD} = \left[\begin{array}{c|c} \mathbf{F}_{DR} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{F}_{RS} \end{array}\right] = \left[\begin{array}{cc|cc} N_r & N_\beta & 0 & 0 \\ \left(\frac{Y_r}{V_N} - 1\right) & \frac{Y_\beta}{V_N} & 0 & 0 \\ \hline 0 & 0 & L_p & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\mathbf{G}_{LD} = \left[\begin{array}{c|c} N_{\delta R} & 0 \\ \frac{Y_{\delta R}}{V_N} & 0 \\ \hline 0 & L_{\delta A} \\ 0 & 0 \end{array}\right]$$

$$\mathbf{L}_{LD} = \left[\begin{array}{c|c} N_\beta & 0 \\ \frac{Y_\beta}{V_N} & 0 \\ \hline 0 & L_p \\ 0 & 0 \end{array}\right]$$

31

2nd-Order Models of Lateral-Directional Motion

Approximate Dutch Roll Equation

$$\Delta\dot{\mathbf{x}}_{DR} = \begin{bmatrix} \Delta\dot{r} \\ \Delta\dot{\beta} \end{bmatrix} \approx \begin{bmatrix} N_r & N_\beta \\ \left(\frac{Y_r}{V_N} - 1\right) & \frac{Y_\beta}{V_N} \end{bmatrix} \begin{bmatrix} \Delta r \\ \Delta\beta \end{bmatrix} + \begin{bmatrix} N_{\delta R} \\ \frac{Y_{\delta R}}{V_N} \end{bmatrix} \Delta\delta R + \begin{bmatrix} N_\beta \\ \frac{Y_\beta}{V_N} \end{bmatrix} \Delta\beta_{wind}$$

Approximate Spiral-Roll Equation

$$\Delta\dot{\mathbf{x}}_{RS} = \begin{bmatrix} \Delta\dot{p} \\ \Delta\dot{\phi} \end{bmatrix} \approx \begin{bmatrix} L_p & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta p \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} L_{\delta A} \\ 0 \end{bmatrix} \Delta\delta A + \begin{bmatrix} L_p \\ 0 \end{bmatrix} \Delta p_{wind}$$

32

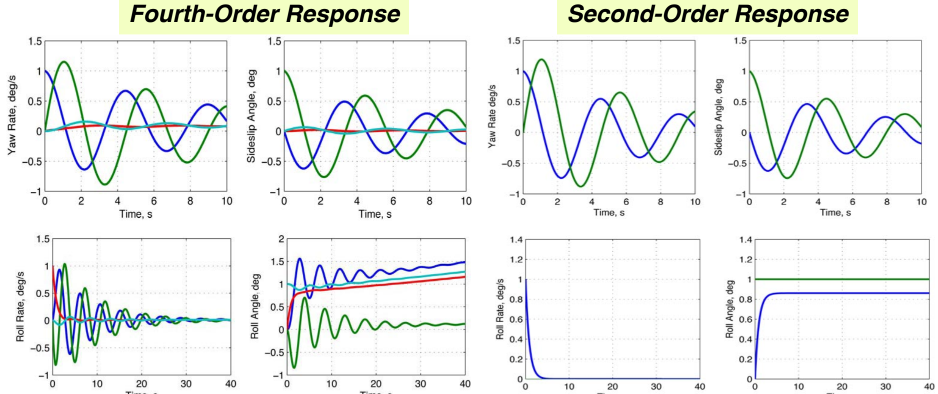
Comparison of 4^{th}- and 2^{nd}-Order Dynamic Models

33

Comparison of 2^{nd}- and 4^{th}-Order Initial-Condition Responses of Business Jet

4 initial conditions [$r(0)$, $\beta(0)$, $p(0)$, $\phi(0)$]

Fourth-Order Response ***Second-Order Response***



Speed and damping of responses is adequately portrayed by 2^{nd}-order models
Roll-spiral modes have little effect on yaw rate and sideslip angle responses
BUT Dutch roll mode has large effect on roll rate and roll angle responses

34

Next Time: Analysis of Time Response

Reading:
Flight Dynamics
298-313, 338-342

Learning Objectives

- **Methods of time-domain analysis**
  - **Continuous- and discrete-time models**
  - **Transient response to initial conditions and inputs**
  - **Steady-state (equilibrium) response**
  - **Phase-plane plots**
  - **Response to sinusoidal input**

35

Supplemental Material

36

Side Velocity Dynamics

Nonlinear equation

$$\dot{v} = Y_B / m + g\sin\phi\cos\theta_N - ru_N + pw_N$$

First row of linearized dynamic equation

$$\begin{aligned}\Delta\dot{v}(t) = &\left[F_{11}\Delta v(t) + F_{12}\Delta p(t) + F_{13}\Delta r(t) + F_{14}\Delta\phi(t)\right]\\ &+\left[G_{11}\Delta\delta A(t) + G_{12}\Delta\delta R(t)\right]\\ &+\left[L_{11}\Delta v_{wind} + L_{12}\Delta p_{wind}\right]\end{aligned}$$

37

Side Velocity Sensitivity to State Perturbations

$$\dot{v} = Y_B / m + g\sin\phi\cos\theta_N - ru_N + pw_N$$

Coefficients in first row of F

$$F_{11} = \frac{1}{m}\left(C_{Y_v}\frac{\rho_N V_N^2}{2}S\right) \triangleq Y_v$$

$$F_{12} = \frac{1}{m}\left(C_{Y_p}\frac{\rho_N V_N^2}{2}S\right) + w_N \triangleq Y_p + w_N$$

$$F_{13} = \frac{1}{m}\left(C_{Y_r}\frac{\rho_N V_N^2}{2}S\right) - u_N \triangleq Y_r - u_N$$

$$F_{14} = g\cos\phi\cos\alpha_N \approx g$$

38

Side Velocity Sensitivity to Control and Disturbance Perturbations

Coefficients in first rows of G and L

$$G_{11} = \frac{1}{m}\left[C_{Y_{\delta A}} \frac{\rho_N V_N^2}{2} S \right] \triangleq Y_{\delta A}$$

$$G_{12} = \frac{1}{m}\left[C_{Y_{\delta R}} \frac{\rho_N V_N^2}{2} S \right] \triangleq Y_{\delta R}$$

$$L_{11} = -F_{11}$$
$$L_{12} = -F_{12}$$

39

Roll Rate Dynamics

Nonlinear equation

$$\dot{p} = \left(I_{zz} L_B + I_{xz} N_B \right) \Big/ \left(I_{xx} I_{zz} - I_{xz}^2 \right)$$

Second row of linearized dynamic equation

$$\begin{aligned} \Delta\dot{p}(t) = &\left[F_{21}\Delta v(t) + F_{22}\Delta p(t) + F_{23}\Delta r(t) + F_{24}\Delta\phi(t) \right] \\ &+\left[G_{21}\Delta\delta A(t) + G_{22}\Delta\delta R(t) \right] \\ &+\left[L_{21}\Delta v_{wind} + L_{22}\Delta p_{wind} \right] \end{aligned}$$

40

Roll Rate Sensitivity to State Perturbations

$$\dot{p} = \left(I_{zz}L_B + I_{xz}N_B\right) / \left(I_{xx}I_{zz} - I_{xz}^2\right)$$

Coefficients in second row of F

$$F_{21} = \left(I_{zz}\frac{\partial L_B}{\partial v} + I_{xz}\frac{\partial N_B}{\partial v}\right) / \left(I_{xx}I_{zz} - I_{xz}^2\right) \triangleq L_v$$

$$F_{22} = \left(I_{zz}\frac{\partial L_B}{\partial p} + I_{xz}\frac{\partial N_B}{\partial p}\right) / \left(I_{xx}I_{zz} - I_{xz}^2\right) \triangleq L_p$$

$$F_{23} = \left(I_{zz}\frac{\partial L_B}{\partial r} + I_{xz}\frac{\partial N_B}{\partial r}\right) / \left(I_{xx}I_{zz} - I_{xz}^2\right) \triangleq L_r$$

$$F_{24} = 0$$

41

Yaw Rate Dynamics

Nonlinear equation

$$\dot{r} = \left(I_{xz}L_B + I_{xx}N_B\right) / \left(I_{xx}I_{zz} - I_{xz}^2\right)$$

Third row of linearized dynamic equation

$$\begin{aligned}\Delta\dot{r}(t) = &\left[F_{31}\Delta v(t) + F_{32}\Delta p(t) + F_{33}\Delta r(t) + F_{34}\Delta\phi(t)\right] \\ &+\left[G_{31}\Delta\delta A(t) + G_{32}\Delta\delta R(t)\right] \\ &+\left[L_{31}\Delta v_{wind} + L_{32}\Delta p_{wind}\right]\end{aligned}$$

42

Yaw Rate Sensitivity to State Perturbations

$$\dot{r} = \left(I_{xz}L_B + I_{xx}N_B\right) \Big/ \left(I_{xx}I_{zz} - I_{xz}^2\right)$$

Coefficients in third row of F

$$F_{31} = \left(I_{xz}\frac{\partial L_B}{\partial v} + I_{xx}\frac{\partial N_B}{\partial v}\right) \Big/ \left(I_{xx}I_{zz} - I_{xz}^2\right) \triangleq N_v$$

$$F_{32} = \left(I_{xz}\frac{\partial L_B}{\partial p} + I_{xx}\frac{\partial N_B}{\partial p}\right) \Big/ \left(I_{xx}I_{zz} - I_{xz}^2\right) \triangleq N_p$$

$$F_{33} = \left(I_{xz}\frac{\partial L_B}{\partial r} + I_{xx}\frac{\partial N_B}{\partial r}\right) \Big/ \left(I_{xx}I_{zz} - I_{xz}^2\right) \triangleq N_r$$

$$F_{34} = 0$$

43

Roll Angle Dynamics

Nonlinear equation

$$\dot{\phi} = p + \left(r\cos\phi\right)\tan\theta_N$$

Fourth row of linearized dynamic equation

$$\begin{aligned}\Delta\dot{\phi}(t) = &\left[F_{41}\Delta v(t) + F_{42}\Delta p(t) + F_{43}\Delta r(t) + F_{44}\Delta\phi(t)\right] \\ &+ \left[G_{41}\Delta\delta A(t) + G_{42}\Delta\delta R(t)\right] \\ &+ \left[L_{41}\Delta v_{wind} + L_{42}\Delta p_{wind}\right]\end{aligned}$$

44

Roll Angle Sensitivity to State Perturbations

$$\dot{\phi} = p + (r\cos\phi)\tan\theta_N$$

Coefficients in fourth row of F

$$F_{41} = 0 \qquad F_{43} = \cos\phi_N \tan\theta_N \approx \tan\theta_N$$

$$F_{42} = 1 \qquad F_{44} = r_N \sin\phi_N \tan\theta_N = 0$$

45

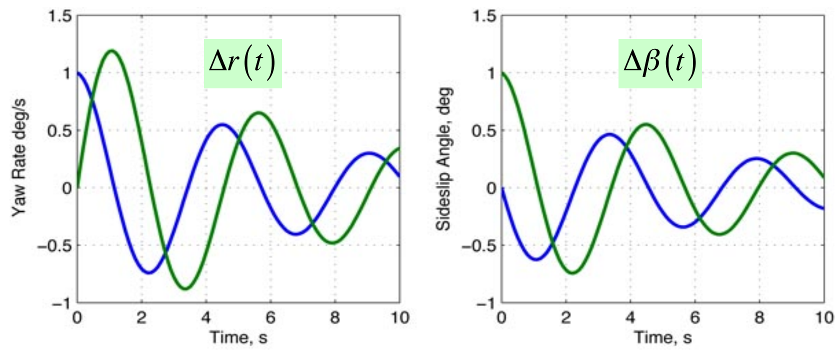
Primary Lateral-Directional Control Derivatives

$$L_{\delta A} = C_{l_{\delta A}} \left(\frac{\rho_N V_N^2}{2I_{xx}} \right) Sb$$

$$N_{\delta R} = C_{n_{\delta R}} \left(\frac{\rho_N V_N^2}{2I_{zz}} \right) Sb$$

46

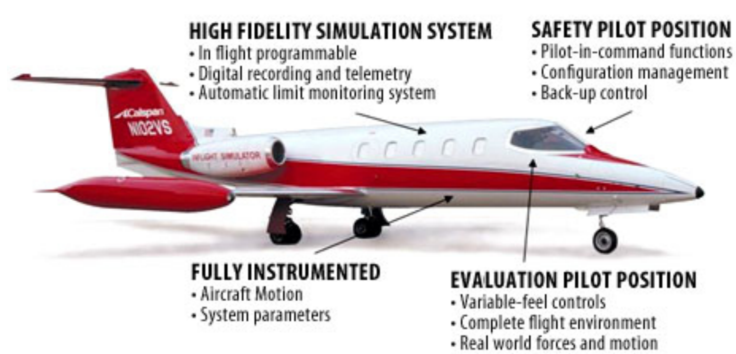
Initial Condition Response of Approximate Dutch Roll Mode



47

Dutch roll Video

Calspan Variable-Stability Learjet Dutch roll Oscillation
http://www.youtube.com/watch?v=1_TW9oz99NQ



48