

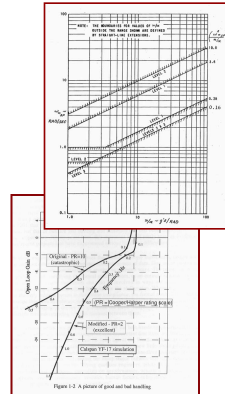
# Flying Qualities Criteria

Robert Stengel, Aircraft Flight Dynamics  
MAE 331, 2018

## Learning Objectives

- MIL-F-8785C criteria
- CAP, C\*, and other longitudinal criteria
- $\phi/\beta$ ,  $\omega_\phi/\omega_d$ , and other lateral-directional criteria
- Pilot-vehicle interactions
- Flight control system design

*Flight Dynamics*  
419-428, 525-533, 624-629  
*Airplane Stability and Control*  
Chapter 21



Copyright 2018 by Robert Stengel. All rights reserved. For educational use only.  
<http://www.princeton.edu/~stengel/MAE331.htm>  
<http://www.princeton.edu/~stengel/FlightDynamics.html>

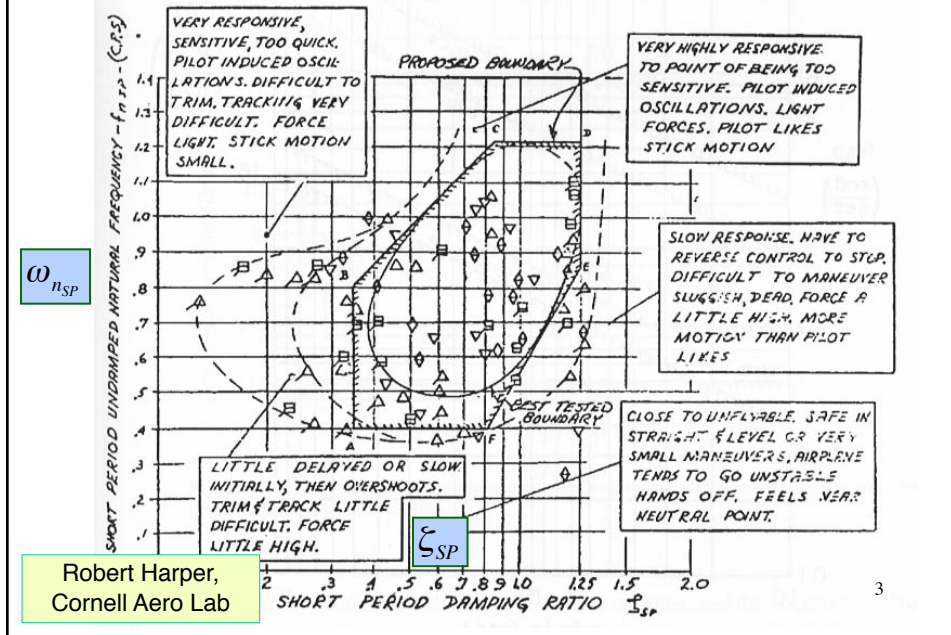
1

## Design for Satisfactory Flying Qualities

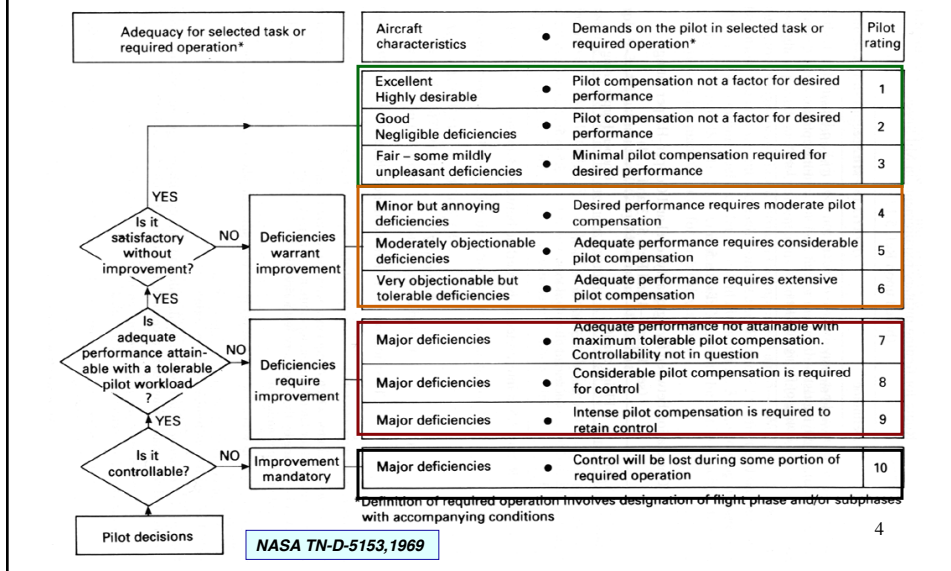
- Satisfy procurement requirement (e.g., Mil Standard)
- Satisfy test pilots (e.g., *Cooper-Harper ratings*)
- Avoid pilot-induced oscillations (PIO)
- Minimize time-delay effects
- Time- and frequency-domain criteria

2

## Short-Period "Bullseye" or "Thumbprint"



## Cooper-Harper Handling Qualities Rating Scale

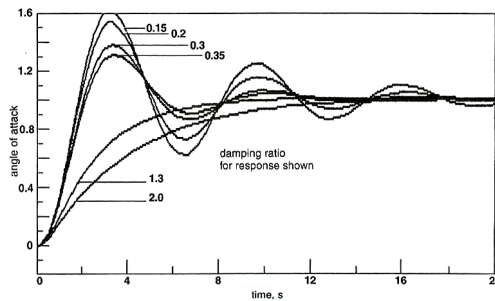


## MIL-F-8785C Identifies Satisfactory, Acceptable, and Unacceptable Response Characteristics

### Damping Ratio

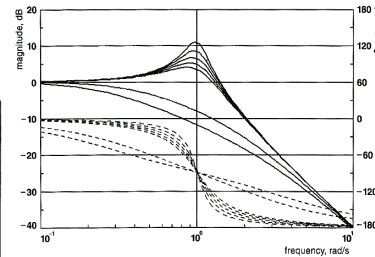
Level	Category A and C Flight Phases		Category B Flight Phases	
	Minimum	Maximum	Minimum	Maximum
1	0.35	1.30	0.30	2.00
2	0.25	2.00	0.20	2.00
3	0.15*	-	0.15	-

\* May be reduced at altitudes above 20 000 feet if approved by the procuring activity.



Step Response

### Short-period angle-of-attack response to elevator input



Frequency Response

5

## Military Flying Qualities Specifications, MIL-F-8785C

- Specifications established during WWII
- US Air Force and Navy coordinated efforts beginning in 1945
- First version appeared in 1948, last in 1980
- Distinctions by flight phase, mission, and aircraft type
- Replaced by **Military Flying Qualities Standard, MIL-STD-1797A**, with procurement-specific criteria
- MIL-F-8785C no longer “official,” but still a good reference for analysis

6

## MIL-F-8785C Aircraft Types

- I. **Small, light airplanes, e.g., utility aircraft and primary trainers**
- II. **Medium-weight, low-to-medium maneuverability airplanes, e.g., small transports or tactical bombers**
- III. **Large, heavy, low-to-medium maneuverability airplanes, e.g., heavy transports, tankers, or bombers**
- IV. **Highly maneuverable aircraft, e.g., fighter and attack airplanes**

7

## MIL-F-8785C Flight Phase

- A. **Non-terminal flight requiring rapid maneuvering precise tracking, or precise flight path control**
  - air-to-air combat
  - ground attack
  - in-flight refueling (receiver)
  - close reconnaissance
  - terrain following
  - close formation flying
- B. **Non-terminal flight requiring gradual maneuvering**
  - climb, cruise
  - in-flight refueling (tanker)
  - descent
- C. **Terminal flight**
  - takeoff (normal and catapult)
  - approach
  - wave-off/go-around
  - landing

8

## MIL-F-8785C Levels of Performance

1. Flying qualities clearly adequate for the mission flight phase
2. Flying qualities adequate to accomplish the mission flight phase, with some increase in pilot workload or degradation of mission effectiveness
3. Flying qualities such that the aircraft can be controlled safely, but pilot workload is excessive or mission effectiveness is inadequate

9

## Principal MIL-F-8785C Metrics

- **Longitudinal flying qualities**
  - static speed stability
  - phugoid stability
  - flight path stability
  - short period frequency and its relationship to command acceleration sensitivity
  - short period damping
  - control-force gradients
- **Lateral-directional flying qualities**
  - natural frequency and damping of the Dutch roll mode
  - time constants of the roll and spiral modes
  - rolling response to commands and Dutch roll oscillation
  - sideslip excursions
  - maximum stick and pedal forces
  - turn coordination

10

# *Longitudinal Criteria*

11

## Long-Period Flying Qualities Criteria (MIL-F-8785C)

### Flight Phase

- A. Non-terminal flight requiring rapid maneuvering
- B. Non-terminal flight requiring gradual maneuvering
- C. Terminal flight

### Level of Performance

- 1. Clearly adequate for the mission
- 2. Adequate to accomplish the mission, with some increase in workload
- 3. Aircraft can be controlled safely, but workload is excessive

- **Static speed stability**

- **No tendency for aperiodic divergence**
  - **Phugoid oscillation -> 2 real roots, 1 that is unstable**
- **Stable control stick position and force gradients**
  - e.g., Increasing “pull” position and force with decreasing speed

12

## Long-Period Flying Qualities Criteria (MIL-F-8785C)

### Steady-State Response to Elevator

$$\Delta V_{SS} = a \Delta \delta E_{SS}$$

$$\Delta \gamma_{SS} = c \Delta \delta E_{SS}$$

Ratio

$$\frac{\Delta \gamma_{SS}}{\Delta V_{SS}} = \frac{c}{a} \quad (\text{with appropriate scaling})$$

- **Flight path stability [Phase C]**

1.  $(\Delta \gamma / \Delta V)_{SS} < 0.06 \text{ deg/kt}$
2.  $(\Delta \gamma / \Delta V)_{SS} < 0.15 \text{ deg/kt}$
3.  $(\Delta \gamma / \Delta V)_{SS} < 0.24 \text{ deg/kt}$

13

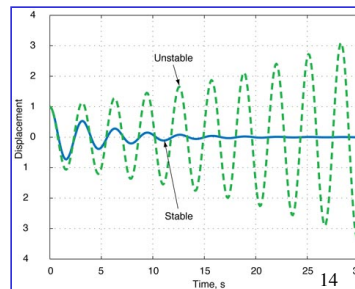
## Long-Period Flying Qualities Criteria (MIL-F-8785C)

- **Phugoid stability**

1. Damping ratio  $\geq 0.04$
2. Damping ratio  $\geq 0$
3. "Time to double",  $T_2 \geq 55 \text{ sec}$

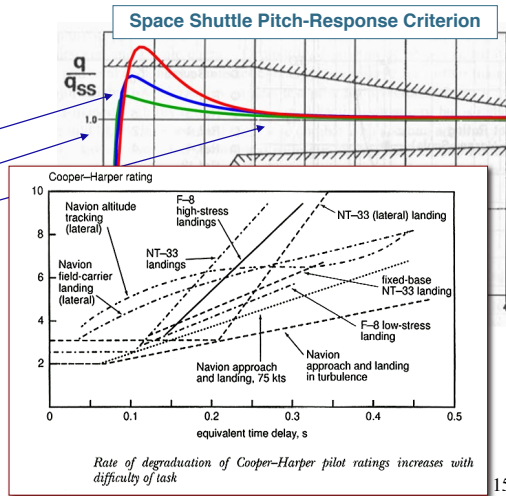
**Time to Double**

$$T_{2_{Ph}} = -0.693 / \zeta_{Ph} \omega_{n_{Ph}}$$



# Short Period Criteria

- **Important parameters**
  - Short-period natural frequency
  - Damping ratio
  - Lift slope
  - Step response
    - Over-/under-shoot
    - Rise time
    - Settling time
    - Pure time delay
  - Pitch angle response
  - Normal load factor response
  - Flight path angle response (landing)

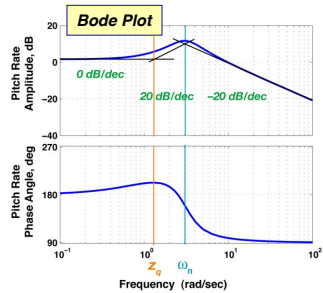


15

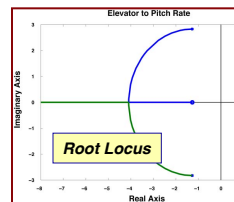
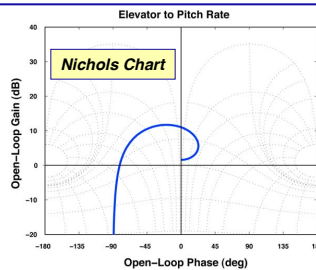
# Short-Period Approximation Transfer Functions



- **Elevator to pitch rate**



$$\frac{\Delta q(s)}{\Delta \delta E(s)} = \frac{k_q (s - z_q)}{s^2 + 2\zeta_{SP} \omega_{nSP} s + \omega_{nSP}^2} = \frac{k_q \left( s + \frac{1}{T_{\theta_2}} \right)}{s^2 + 2\zeta_{SP} \omega_{nSP} s + \omega_{nSP}^2}$$



- Pure gain or phase change (< 90 deg) in feedback control cannot produce instability

16

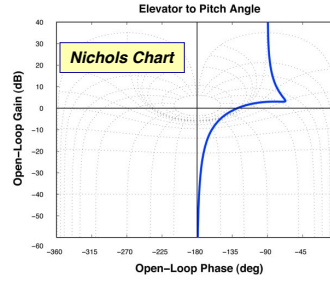
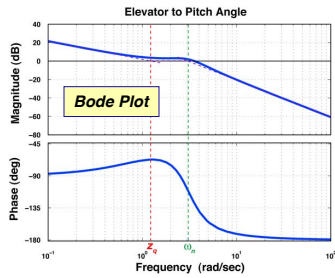




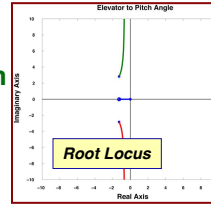
## Short-Period Approximation Transfer Functions

- Elevator to pitch angle
- Integral of prior example

$$\frac{\Delta\theta(s)}{\Delta\delta E(s)} = \frac{k_q (s - z_q)}{s(s^2 + 2\zeta_{SP}\omega_{n_{SP}}s + \omega_{n_{SP}}^2)}$$



- Pure gain or phase change (< 45 deg) in feedback control cannot produce instability



17

## Normal Load Factor

$$\Delta n_z = \frac{V_N}{g} (\Delta\dot{\alpha} - \Delta q) = -\frac{V_N}{g} \left( \frac{L_\alpha}{V_N} \Delta\alpha + \frac{L_{\delta E}}{V_N} \Delta\delta E \right)$$

positive down

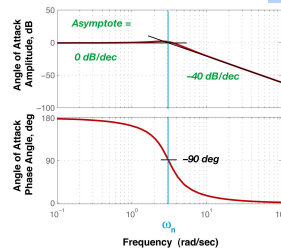
- Therefore, with negligible  $L_{\delta E}$  (aft tail/canard effect)

$$\frac{\partial \Delta n_z(s)}{\partial \Delta\delta E(s)} = \frac{1}{g} \left( L_\alpha \frac{\partial \Delta\alpha(s)}{\partial \Delta\delta E(s)} + L_{\delta E} \right) \approx \left( \frac{L_\alpha}{g} \right) \frac{\partial \Delta\alpha(s)}{\partial \Delta\delta E(s)}$$

positive up

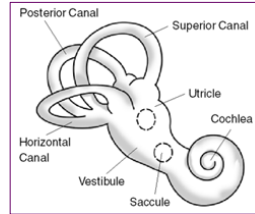
- Elevator to angle of attack ( $L_{\delta E} = 0$ )

$$\frac{\Delta\alpha(s)}{\Delta\delta E(s)} \approx \frac{k_\alpha}{s^2 + 2\zeta_{SP}\omega_{n_{SP}}s + \omega_{n_{SP}}^2}$$



18

## Control Anticipation Parameter, **CAP**



Inner ear senses angular acceleration about 3 axes

**Initial Angular Acceleration**

$$\Delta \dot{q}(0) = \left( M_{\delta E} - \frac{M_{\alpha}}{V_N + L_{\alpha}} L_{\delta E} \right) \Delta \delta E_{SS}$$

**Desired Normal Load Factor**

$$\Delta n_{SS} = \frac{V_N}{g} \Delta q_{SS} = - \left( \frac{V_N}{g} \right) \frac{\left( M_{\delta E} \frac{L_{\alpha}}{V_N} - M_{\alpha} \frac{L_{\delta E}}{V_N} \right)}{\left( M_q \frac{L_{\alpha}}{V_N} + M_{\alpha} \right)} \Delta \delta E_{SS}$$

19

## Control Anticipation Parameter, **CAP**

Inner ear cue should aid pilot in anticipating commanded normal acceleration

$$CAP = \frac{\Delta \dot{q}(0)}{\Delta n_{SS}} = \frac{- \left( M_{\delta E} - \frac{M_{\alpha}}{V_N + L_{\alpha}} L_{\delta E} \right) \left( M_q \frac{L_{\alpha}}{V_N} + M_{\alpha} \right)}{\left( L_{\alpha} M_{\delta E} - L_{\delta E} M_{\alpha} \right) / g}$$

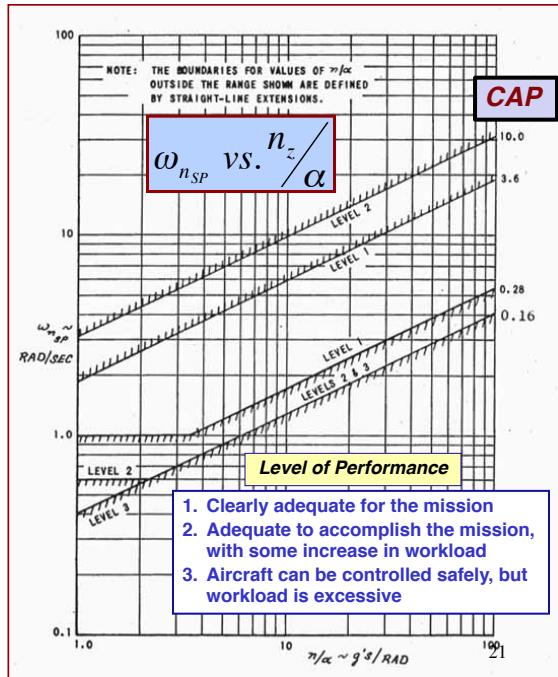
with  $L_{\delta E} = 0$

$$CAP = \frac{- \left( M_q \frac{L_{\alpha}}{V_N} + M_{\alpha} \right)}{L_{\alpha} / g} \approx \frac{\omega_{n_{SP}}^2}{n_z / \alpha}$$

20

# MIL-F-8785C Short-Period Flying Qualities Criterion

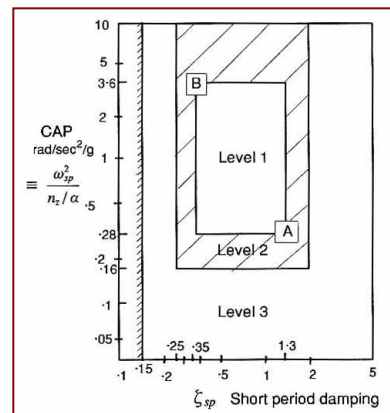
**CAP** =  
constant along  
Level  
Boundaries



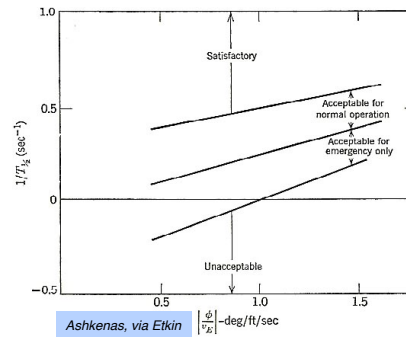
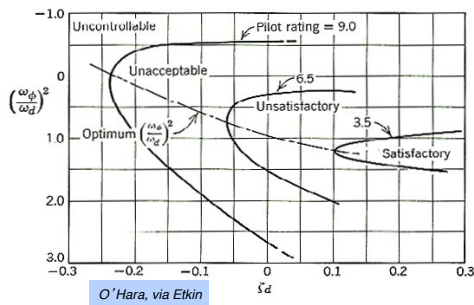
## Control Anticipation Parameter vs. Short-Period Damping Ratio (MIL-F-8785C, Category A)

$$CAP = \frac{-\left(M_q \frac{L_\alpha}{V_N} + M_\alpha\right)}{L_\alpha / g}$$

$$\approx \frac{\omega_{n_{SP}}^2}{n_z / \alpha}$$



## Early Lateral-Directional Flying Qualities Criteria



$$T_{1/2} = 0.693 / \zeta \omega_n$$

$$v = V_N \beta$$

Time to Half

23

## Lateral-Directional Flying Qualities Parameters



- Lateral Control Divergence Parameter, **LCDP**
- $\phi/\beta$  Effect
- $\omega_\phi/\omega_d$  Effect



24

## Lateral Control Divergence Parameter (**LCDP**)

- Aileron deflection produces yawing as well as rolling moment
  - “Favorable yaw” aids the turn command
  - “Adverse yaw” opposes it
- Equilibrium response to constant aileron input

$$\frac{\Delta\phi_s}{\Delta\delta A_s} = \frac{\left(N_\beta + N_r \frac{Y_\beta}{V_N}\right)L_{\delta A} - \left(L_\beta + L_r \frac{Y_\beta}{V_N}\right)N_{\delta A}}{g/V_N (L_\beta N_r - L_r N_\beta)}$$

- Large-enough  $N_{\delta A}$  effect can reverse the sign of the response
  - Can occur at high angle of attack
  - Can cause “departure from controlled flight”
- Lateral Control Divergence Parameter provides simplified criterion

$$\frac{(N_\beta)L_{\delta A} - (L_\beta)N_{\delta A}}{L_{\delta A}} = N_\beta - \frac{N_{\delta A}}{L_{\delta A}}L_\beta$$

$$\text{LCDP} \equiv C_{n_\beta} - \frac{C_{n_{\delta A}}}{C_{l_{\delta A}}} C_{l_\beta}$$

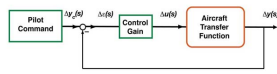
## $\omega_\phi/\omega_d$ Effect



- Aileron-to-roll-angle transfer function

$$\frac{\Delta\phi(s)}{\Delta\delta A(s)} = \frac{k_\phi (s^2 + 2\xi_\phi \omega_\phi s + \omega_\phi^2)}{(s - \lambda_S)(s - \lambda_R)(s^2 + 2\xi_{DR} \omega_{n_{DR}} s + \omega_{n_{DR}}^2)}$$

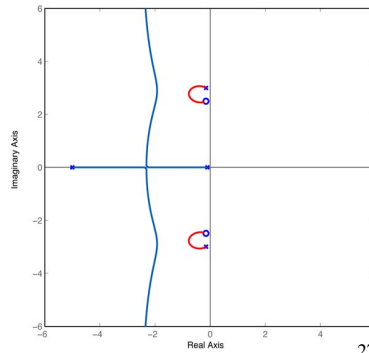
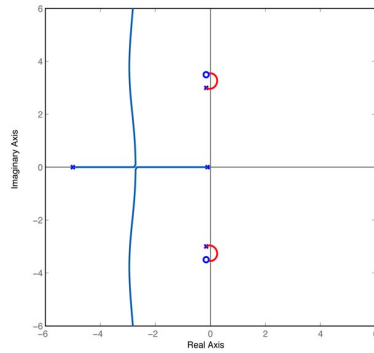
- $\omega_\phi$  is the “natural frequency” of the complex zeros
- $\omega_d = \omega_{n_{DR}}$  is the natural frequency of the Dutch roll mode
- Conditional instability may occur with closed-loop control of roll angle, even with a perfect pilot



## $\omega_\phi/\omega_\delta$ Effect is Important in Roll Angle Control

$$\frac{\Delta\phi(s)}{\Delta\delta A(s)} = \frac{k_\phi (s^2 + 2\zeta_\phi \omega_\phi s + \omega_\phi^2)}{(s - \lambda_S)(s - \lambda_R)(s^2 + 2\zeta_{DR} \omega_{n_{DR}} s + \omega_{n_{DR}}^2)}$$

- As feedback gain increases, Dutch roll roots go to numerator zeros
- If zeros are over poles, **conditional instability** results



27



## $\phi/\beta$ Effect

- $\phi/\beta$  measures the **degree of rolling response in the Dutch roll mode**
  - Large  $\phi/\beta$ : Dutch roll is primarily a rolling motion
  - Small  $\phi/\beta$ : Dutch roll is primarily a yawing motion
- **Eigenvectors,  $e_i$** , indicate the degree of participation of the state component in the  $i^{th}$  mode of motion

$$\det(s\mathbf{I} - \mathbf{F}) = (s - \lambda_1)(s - \lambda_2) \dots (s - \lambda_n)$$

$$(\lambda_i \mathbf{I} - \mathbf{F}) \mathbf{e}_i = \mathbf{0}$$

28

## Eigenvectors

- Eigenvectors,  $\mathbf{e}_i$ , are solutions to the equation

$$(\lambda_i \mathbf{I} - \mathbf{F}) \mathbf{e}_i = \mathbf{0}, \quad i = 1, n$$

or

$$\lambda_i \mathbf{e}_i = \mathbf{F} \mathbf{e}_i, \quad i = 1, n$$

- For each eigenvalue, the corresponding eigenvector can be found (within an arbitrary constant) from

$$\text{Adj}(\lambda_i \mathbf{I} - \mathbf{F}) = \begin{pmatrix} a_1 \mathbf{e}_i & a_2 \mathbf{e}_i & \dots & a_n \mathbf{e}_i \end{pmatrix}, \quad i = 1, n$$

### MATLAB

$$(\mathbf{V}, \mathbf{D}) = \text{eig}(\mathbf{F})$$

**V:** Modal Matrix (i.e., Matrix of Eigenvectors)

**D:** Diagonal Matrix of Corresponding Eigenvalues

29

## $\phi/\beta$ Effect

- With  $\lambda_i$  chosen as a complex root of the Dutch roll mode, the corresponding eigenvector is

$$\mathbf{e}_{DR+} = \begin{bmatrix} e_r \\ e_\beta \\ e_p \\ e_\phi \end{bmatrix}_{DR+} = \begin{bmatrix} (\sigma + j\omega)_r \\ (\sigma + j\omega)_\beta \\ (\sigma + j\omega)_p \\ (\sigma + j\omega)_\phi \end{bmatrix}_{DR+} = \begin{bmatrix} (AR e^{j\phi})_r \\ (AR \phi^{j\phi})_\beta \\ (AR e^{j\phi})_p \\ (AR \phi^{j\phi})_\phi \end{bmatrix}_{DR+}$$

- $\phi/\beta$  is the magnitude of the ratio of the  $\phi$  and  $\beta$  eigenvectors

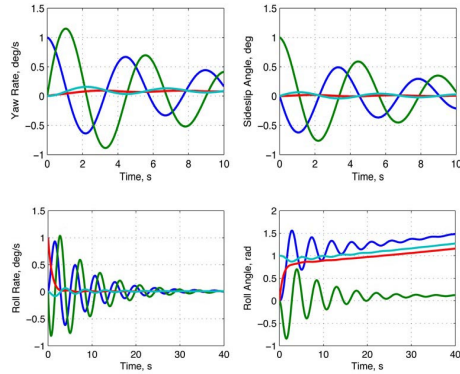
$$\frac{\phi}{\beta} = \left| \frac{(AR)_\phi}{(AR)_\beta} \right| = \left( \frac{V_N}{g} \right) \left[ \left( \zeta_{DR} \omega_{n_{DR}} + \frac{Y_\beta}{V_N} + \frac{L_\beta}{L_r} \right)^2 + \left( \omega_{n_{DR}} \sqrt{1 - \zeta_{DR}^2} \right)^2 \right]^{1/2}$$

30

## $\phi/\beta$ Effect for the Business Jet Example

$$\mathbf{e}_{DR+} = \begin{bmatrix} |e_r| \\ |e_\beta| \\ |e_p| \\ |e_\phi| \end{bmatrix}_{DR+} = \begin{bmatrix} 0.525 \\ 0.416 \\ 0.603 \\ 0.433 \end{bmatrix}_{DR+}$$

$$\frac{\phi}{\beta} = 1.04$$



Roll/Sideslip Angle ratio in the Dutch roll mode

31

## Criteria for Lateral-Directional Modes (MIL-F-8785C)

TABLE VII. Maximum roll-mode time constant, seconds.

Maximum Roll-Mode Time Constant

Flight Phase Category	Class	Level		
		1	2	3
A	I, IV	1.0	1.4	
	II, III	1.4	3.0	
B	All	1.4	3.0	10
C	I, II-C, IV	1.0	1.4	
	II-L, III	1.4	3.0	

TABLE VIII. Spiral stability - minimum time to double amplitude.

Minimum Spiral-Mode Time to Double

Flight Phase Category	Level 1	Level 2	Level 3
A & C	12 sec	8 sec	4 sec
B	20 sec	8 sec	4 sec

32



## Minimum Dutch Roll Natural Frequency and Damping (MIL-F-8785C)

TABLE VI. Minimum Dutch roll frequency and damping.

Flight Phase Level	Category	Class	Min $\zeta_d$ <sup>*</sup>	Min $\omega_{nd}$ <sup>*</sup> rad/sec.	Min $\omega_{nd}$ rad/sec.
1	A (CO and GA)	IV	0.4	-	1.0
	A	I, IV	0.19	0.35	1.0
		II, III	0.19	0.35	0.4 <sup>**</sup>
	B	All	0.08	0.15	0.4 <sup>**</sup>
C	I, II-C, IV	0.08	0.15	1.0	
	II-L, III	0.08	0.10	0.4 <sup>**</sup>	
2	All	All	0.02	0.05	0.4 <sup>**</sup>
3	All	All	0	-	0.4 <sup>**</sup>

\* The governing damping requirement is that yielding the larger value of  $\zeta_d$ , except that a  $\zeta_d$  of 0.7 is the maximum required for Class III.

\*\* Class III airplanes may be excepted from the minimum  $\omega_{nd}$  requirement, subject to approval by the procuring activity, if the requirements of 3.3.2 through 3.3.2.4.1, 3.3.5 and 3.3.9.4 are met. 33

## *Pilot-Vehicle Interactions*

## YF-16 Test Flight Zero

- High-speed taxi test; **no flight intended**
- *Pilot-induced oscillations* from overly sensitive roll control
- Pilot elected to go around rather than eject



35



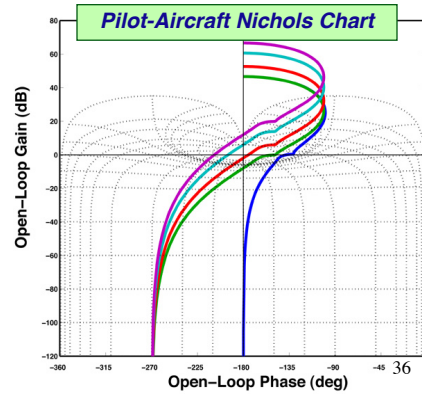
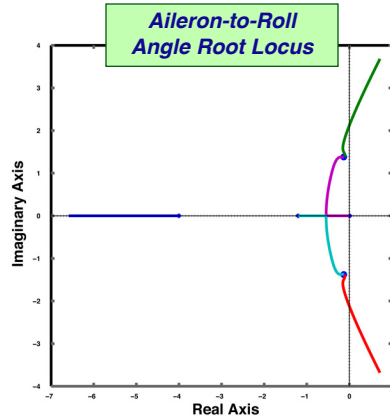
YF-16

## Pilot-Induced Roll Oscillation

Pilot Transfer Function

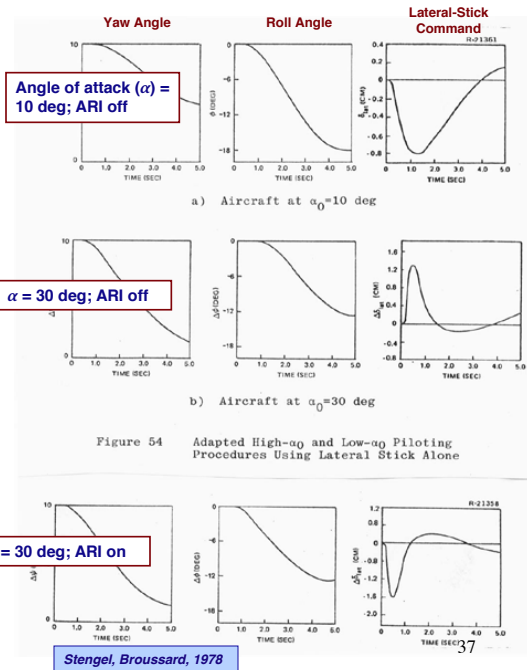
Aircraft Transfer Function

$$\frac{\Delta\phi(s)}{\Delta\delta A(s)}_{\text{pilot in loop}} = \left( \frac{K_p / T_p}{s + 1/T_p} \right) \left[ \frac{k_\phi (s^2 + 2\xi_\phi \omega_\phi s + \omega_\phi^2)}{(s - \lambda_S)(s - \lambda_R)(s^2 + 2\xi_{DR} \omega_{n_{DR}} s + \omega_{n_{DR}}^2)} \right]$$



# Inverse Problem of Lateral Control

- Given a flight path, what is the control history that generates it?
  - Necessary piloting actions
  - Control-law design
- Aileron-rudder interconnect (ARI) simplifies pilot input

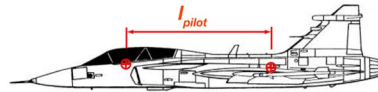


*Next Time:  
Maneuvering at High Angle of  
Attack and Angular Rate*

# SUPPLEMENTAL MATERIAL

39

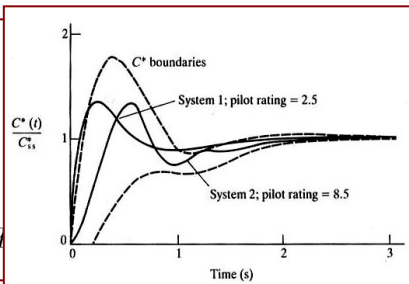
## C\* Criterion



- Hypothesis

- C\* blends normal load factor at pilot's location and pitch rate
- Step response of C\* should lie within acceptable envelope

$$\begin{aligned}
 C^* &= \Delta n_{pilot} + \frac{V_{crossover}}{g} \Delta q \\
 &= \left( l_{pilot} \Delta \dot{q} + \Delta n_{cm} \right) + \frac{V_{crossover}}{g} \Delta q \\
 &= \left[ l_{pilot} \Delta \dot{q} + \frac{V_N}{g} (\Delta q - \Delta \dot{c}) \right] + \frac{V_{crossover}}{g} \Delta q
 \end{aligned}$$



- Below  $V_{crossover}$   $\Delta q$  is pilot's primary control objective
- Above  $V_{crossover}$   $\Delta n_{pilot}$  is the primary control objective

Fighter Aircraft:  $V_{crossover} \approx 125 \text{ m / s}$

40

## Gibson Dropback Criterion for Pitch Angle Control

- Step response of pitch rate should have overshoot for satisfactory pitch and flight path angle response

$$\frac{\Delta q(s)}{\Delta \delta E(s)} = \frac{k_q \left( s + \frac{1}{T_{\theta_2}} \right)}{s^2 + 2\zeta_{SP} \omega_{n_{SP}} s + \omega_{n_{SP}}^2}$$

$$= \frac{k_q \left( s + \frac{\omega_{n_{SP}}}{\zeta_{SP}} \right)}{s^2 + 2\zeta_{SP} \omega_{n_{SP}} s + \omega_{n_{SP}}^2}$$

- Criterion is satisfied when

$$z_{q_1} \triangleq -\frac{1}{T_{\theta_2}} = -\left( \frac{\omega_{n_{SP}}}{\zeta_{SP}} \right)$$

Gibson, 1997

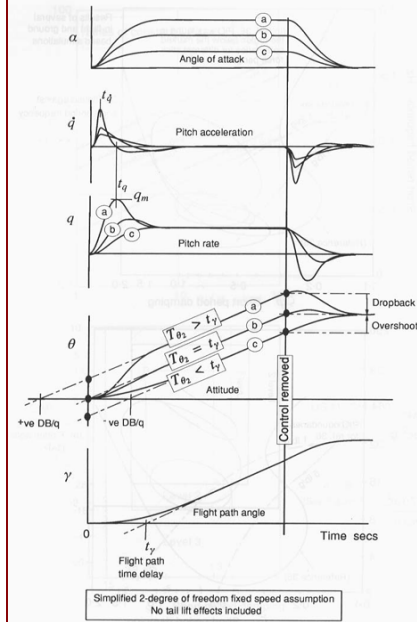


Figure 7-3 Generic short period pitch transient response 41

## Large Aircraft Flying Qualities

- High wing loading, **W/S**
- Distance from pilot to rotational center
- Slosh susceptibility of large tanks
- High wing span -> short relative tail length
  - Higher trim drag
  - Increased yaw due to roll, need for rudder coordination
  - Reduced rudder effect
- Altitude response during approach
  - Increased non-minimum-phase delay in response to elevator
  - Potential improvement from canard
- Longitudinal dynamics
  - Phugoid/short-period resonance
- Rolling response (e.g., time to bank)
- Reduced static stability
- Off-axis passenger comfort in BWB turns



## Criteria for Oscillations and Excursions (MIL-F-8785C)

3.3.2.2 Roll rate oscillations. Following a yaw-control-free step roll control command, the roll rate at the first minimum following the first peak shall be of the same sign and not less than the following percentage of the roll rate at the first peak:

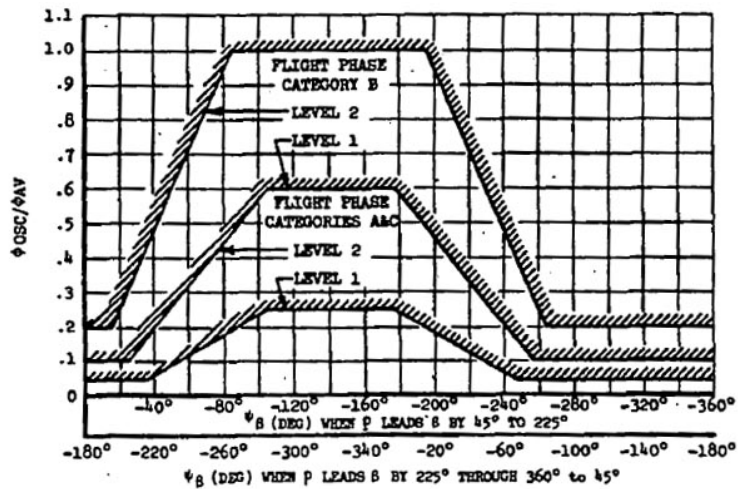
Level	Flight Phase Category	Percent
1	A & C	60
	B	25
2	A & C	25
	B	0

3.3.2.4 Sideslip excursions. Following a yaw-control-free step roll control command, the ratio of the sideslip increment,  $\Delta\beta$ , to the parameter  $k$  (6.2.6) shall be less than the values specified herein. The roll command shall be held fixed until the bank angle has changed at least 90 degrees.

Level	Flight Phase Category	Adverse Sideslip (Right roll command causes right sideslip)	Proverse Sideslip (Right roll command causes left sideslip)
1	A	6 degrees	2 degrees
	B & C	10 degrees	3 degrees
2	All	15 degrees	4 degrees

43

## Criteria for Oscillations and Excursions (MIL-F-8785C)



44

## **Flight Testing Videos**

### ***TSR2 Test Flight***

<http://www.youtube.com/watch?v=GXdJxjvQZW4>

### ***Neil Armstrong, Test Pilot***

<http://www.youtube.com/watch?v=t6DdlPoPOE4>

### ***NASA Dryden (now Armstrong) Flight Research Center***

<http://www.youtube.com/watch?v=j85jlc1Zfk4>

### ***Avro Arrow Revisited***

<https://www.youtube.com/watch?v=S74zf0YZX20>