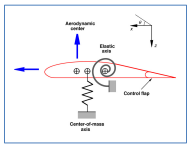
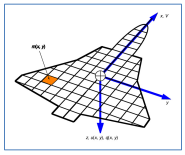
Aeroelasticity and Fuel Slosh

Robert Stengel, Aircraft Flight Dynamics
MAE 331, 2018

Learning Objectives

- Aerodynamic effects of bending and torsion
- Modifications to aerodynamic coefficients
- Dynamic coupling
- Fuel shift and sloshing dynamics





Flight Dynamics
418–419, 549–569, 665–678
Airplane Stability and Control
Chapter 19

http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

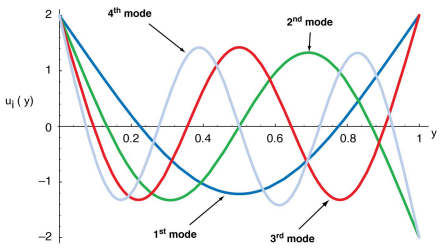
1

Bending Vibrations of a Free-Free Uniform Beam

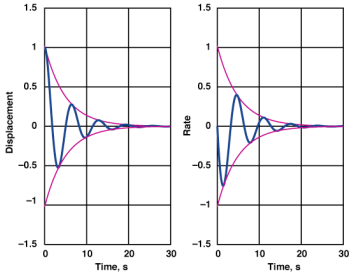
$$EI_y \frac{d^4 z}{dx^4} = k_i = -m' \frac{d^2 z}{dt^2}$$

- In figure, ($u = z$, $y = x$)
- Left side determines vibrational mode shape
- Right side describes oscillation
- Natural frequency of each mode proportional to $(k_i)^{1/2}$

Mode shapes of bending vibrations



Time response of single mode

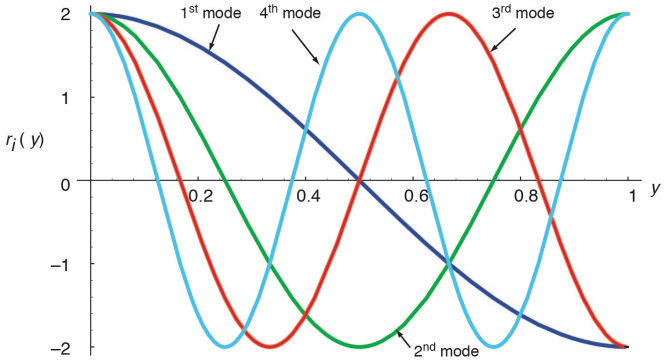


2

Torsional Vibrations of a Free-Free Uniform Beam

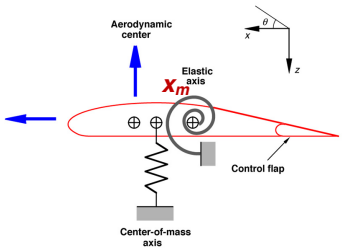
$$GJ\,\theta''\Big|_{y=y_s} = I'_{yy}\ddot{\theta}\Big|_{y=y_s}$$

Mode shapes of torsional vibrations



3

One-Dimensional Model of Wing Aeroelasticity



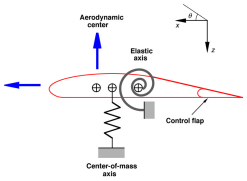
- Wing cross section
- ***z*** due to bending, ***θ*** due to torsion
- ***m'*** = mass per unit length
- ***I'yy*** = inertia per unit length
- ***k, c*** = spring constants
- ***xm*** = axis offset
- Aircraft motion neglected
- 4th-order system

$$k\,z = -m'\ddot{z} - x_m m'\ddot{\theta} - \left[\frac{\partial C_L}{\partial\alpha}\left(\dot{z}/V + \theta\right) + \frac{\partial C_L}{\partial\delta}\delta\right]\bar{q}\,\bar{c}$$

$$-c\,\theta = I'_{yy}\ddot{\theta} + x_m m'\ddot{z} - \left[\frac{\partial C_m}{\partial\alpha}\left(\dot{z}/V + \theta\right) + \frac{\partial C_m}{\partial q}\dot{\theta} + \frac{\partial C_m}{\partial\delta}\delta\right]\bar{q}\,\bar{c}^2$$

4

One-Dimensional Model of Wing Aeroelasticity



$$\begin{bmatrix} \Delta\dot{\mathbf{x}}_{bending} \\ \Delta\dot{\mathbf{x}}_{torsion} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{F}_{bending} & \mathbf{F}_{torsion}^{bending} \\ \hline \mathbf{F}_{bending}^{torsion} & \mathbf{F}_{torsion} \end{array}\right] \begin{bmatrix} \Delta\mathbf{x}_{bending} \\ \Delta\mathbf{x}_{torsion} \end{bmatrix} + \cdots$$

$$\begin{bmatrix} \Delta\mathbf{x}_{bending} \\ \Delta\mathbf{x}_{torsion} \end{bmatrix} = \begin{bmatrix} \Delta z \\ \Delta\dot{z} \\ \Delta\theta \\ \Delta\dot{\theta} \end{bmatrix}$$

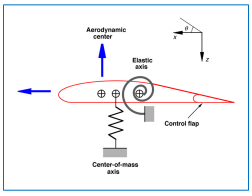
$$\Delta(s) = \left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{bending} \left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{torsion}$$

If $x_m = 0$

$$\begin{bmatrix} \Delta\dot{\mathbf{x}}_{bending} \\ \Delta\dot{\mathbf{x}}_{torsion} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{F}_{bending} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{F}_{torsion} \end{array}\right] \begin{bmatrix} \Delta\mathbf{x}_{bending} \\ \Delta\mathbf{x}_{torsion} \end{bmatrix} + \cdots$$

5

Torsional Divergence



$$\Delta_{torsion}(s) = \left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{torsion}$$

$x_{aero} \simeq$ quarter chord; $x_{elastic} \simeq$ half chord; $\therefore \partial C_m / \partial\alpha > 0$

$$\text{If } \left(\omega_n^{\,2}\right)_{torsion} = \sqrt{\left[c - \frac{\partial C_m}{\partial\alpha}\left(\frac{\rho\bar{c}^2}{2}\right)V^2\right] \Big/ I'_{yy}} < 0,$$

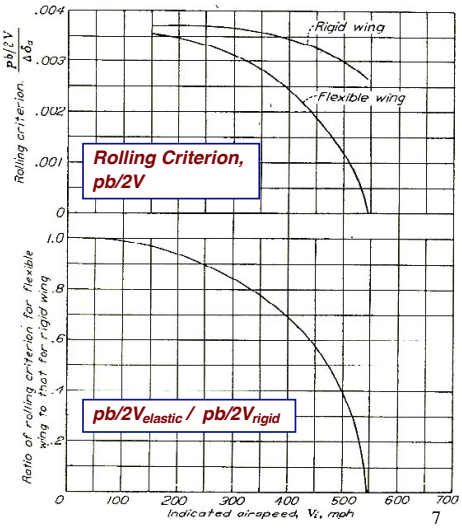
two real roots, and one is unstable

$$V_{torsional\ divergence} = c \Big/ \frac{\partial C_m}{\partial\alpha}\left(\frac{\rho\bar{c}^2}{2}\right)$$

6

Reduced Aileron Effect Due to Aeroelasticity

- Wing torsion reduces aileron effect with increasing dynamic pressure



7

Aeroelastic Aileron Effect of Boeing 2707-300 Supersonic Transport Concept

8

Quasi-Static Aeroelastic Model of Aircraft Dynamics: Residualization

- IF elastic modes are fast compared to rigid modes and are stable

$$\begin{bmatrix} \Delta\dot{\mathbf{x}}_{aircraft} \\ \mathbf{0} \end{bmatrix} \approx \left[\begin{array}{c|c} \mathbf{F}_{aircraft} & \mathbf{F}_{elastic}^{aircraft} \\ \hline \mathbf{F}_{aircraft}^{elastic} & \mathbf{F}_{elastic} \end{array}\right] \begin{bmatrix} \Delta\mathbf{x}_{aircraft} \\ \Delta\mathbf{x}_{elastic} \end{bmatrix} + \begin{bmatrix} \mathbf{G}_{aircraft} \\ \mathbf{G}_{elastic} \end{bmatrix} \Delta\mathbf{u}_{aircraft}$$

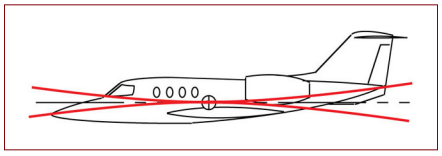
- Residualization reduces aeroelastic model order to rigid-body model order

$$\begin{aligned} \Delta\dot{\mathbf{x}}_a &= \mathbf{F}_a\Delta\mathbf{x}_a - \mathbf{F}_e^a\mathbf{F}_e^{-1}\left[\mathbf{F}_a^e\Delta\mathbf{x}_a + \mathbf{G}_e\Delta\mathbf{u}_a\right] + \mathbf{G}_a\Delta\mathbf{u}_a \\ &= \mathbf{F}'_a\,\Delta\mathbf{x}_a + \mathbf{G}'_a\,\Delta\mathbf{u}_a \end{aligned}$$

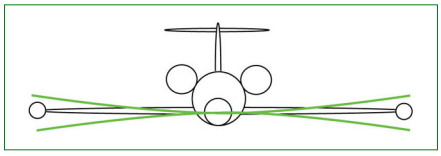
9

Primary Longitudinal Aeroelastic Mode Shapes

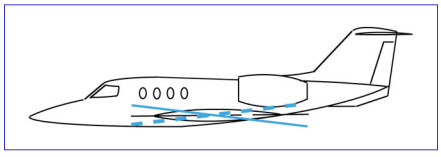
Fuselage Bending



Wing Bending



Wing Torsion



10

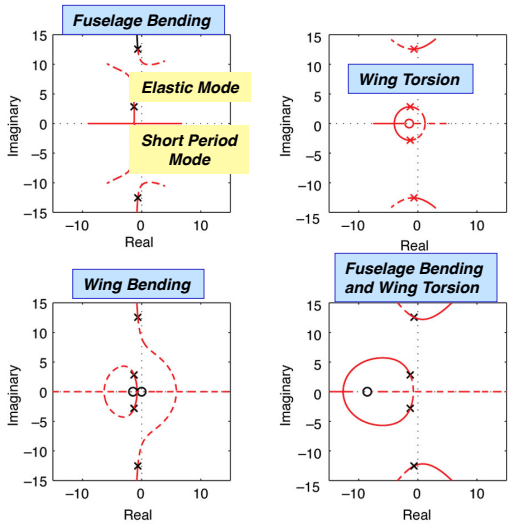
Aeroelastic Model of Aircraft Dynamics

- Coupled model of rigid-body and elastic dynamics

$$\begin{bmatrix} \Delta\dot{\mathbf{x}}_{aircraft} \\ \Delta\dot{\mathbf{x}}_{elastic} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{F}_{aircraft} & \mathbf{F}_{elastic}^{aircraft} \\ \hline \mathbf{F}_{aircraft}^{elastic} & \mathbf{F}_{elastic} \end{array}\right] \begin{bmatrix} \Delta\mathbf{x}_{aircraft} \\ \Delta\mathbf{x}_{elastic} \end{bmatrix} + \begin{bmatrix} \mathbf{G}_{aircraft} \\ \mathbf{G}_{elastic} \end{bmatrix} \Delta\mathbf{u}_{aircraft}$$

11

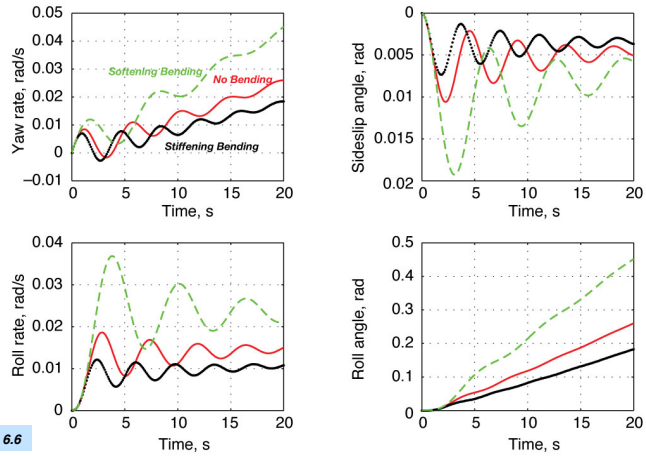
Effect of Increasing Coupling of Single Aeroelastic Mode with Short Period Roots



Flight Dynamics, 5.6

12

Effects of Fuselage Aeroelasticity on Lateral-Directional Response to Rudder Step Input

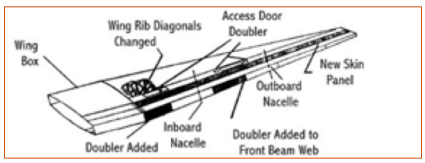


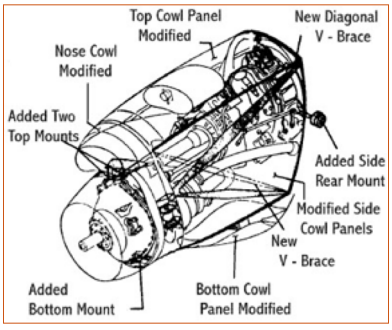
Flight Dynamics, 6.6

13

Aeroelastic Problems of the Lockheed Electra

- **Prop-whirl flutter, 2 fatal accidents (1959-60)**
- **Structural modifications made; aircraft remained in service until 1992**
- **Predecessor of *US Navy Orion P-3*, still in service**

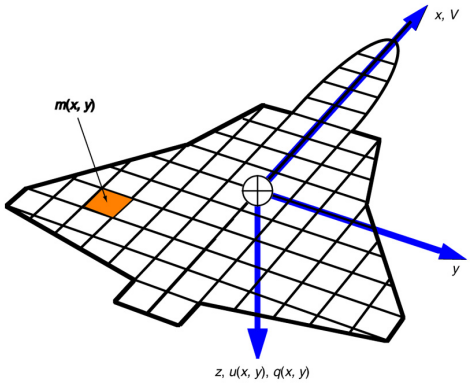




http://www.youtube.com/watch?v=d0fFNWANK5M

14

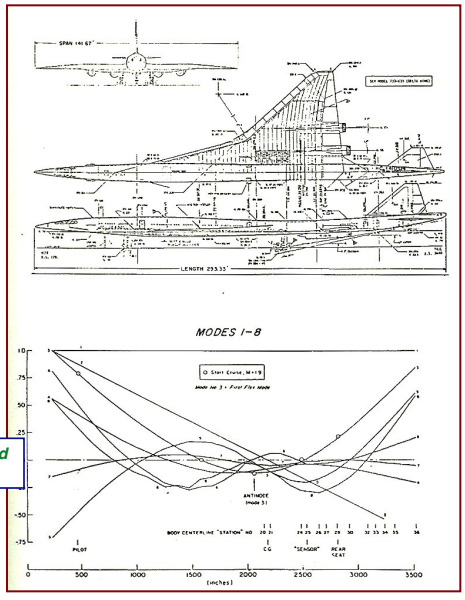
Two-Dimensional Model of Aeroelastic Airplane



Section 4.6, *Flight Dynamics*

15

Longitudinal Structural Modes of Boeing 2707-300 Supersonic Transport Concept



16

Vibrational Mode Shapes for the X-30 (NASP) Vehicle

Computational Grid for Finite-Element Model

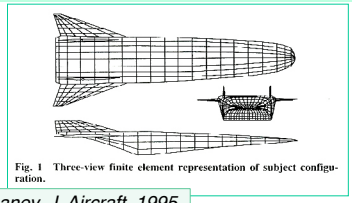


Fig. 1 Three-view finite element representation of subject configuration.

Raney, J. Aircraft, 1995

Shapes of the First Seven Modes

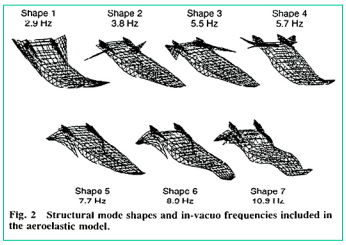


Fig. 2 Structural mode shapes and in-vacuo frequencies included in the aeroelastic model.

Body elastic deflection distorts the shape of scramjet inlet and exhaust ramps

Aeroelastic-propulsive interactions

Impact on flight dynamics

17

B-1 Canards for Ride Control

- **Elastic modes cause severe, high-*g* cockpit vibration during low-altitude, high-speed flight**
- **Active canard surfaces reduce amplitude of the oscillations**

18

Ultra-Light Aircraft

- Extreme aeroelasticity
- *AeroVironment Pathfinder*, *Centurion*, *PathfinderPlus* (solar-electric)
- *Helios* in turbulence

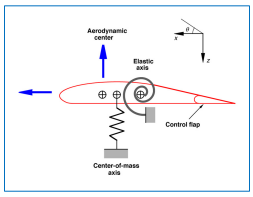
19

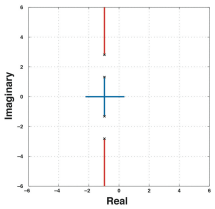
Flutter

If $x_m \neq 0$

$$\begin{bmatrix} \Delta\dot{\mathbf{x}}_{bending} \\ \Delta\dot{\mathbf{x}}_{torsion} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{F}_{bending} & \mathbf{F}^{bending}_{torsion} \\ \hline \mathbf{F}^{torsion}_{bending} & \mathbf{F}_{torsion} \end{array}\right] \begin{bmatrix} \Delta\mathbf{x}_{bending} \\ \Delta\mathbf{x}_{torsion} \end{bmatrix} + \cdots$$

- With increasing x_m, total damping ~ constant but redistributed between modes





- ... plus velocity, air density, nonlinear effects, e.g., cubic spring, damping, ...

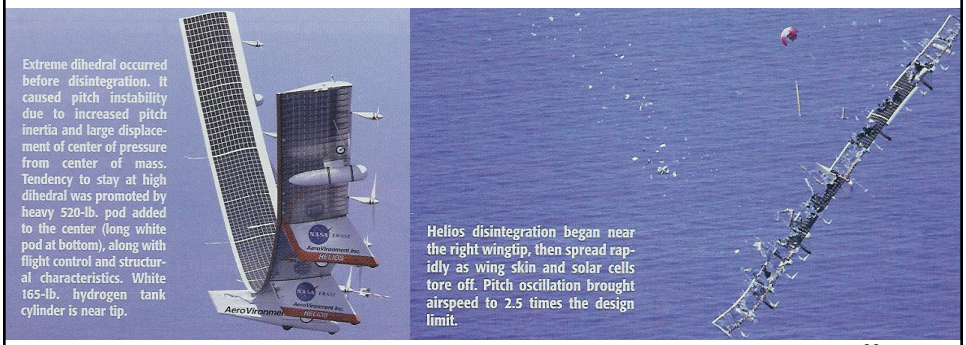
20

Aeroelastic Oscillations

21

The Last Flight of *Helios*

- June 6, 2003
- 2,320 lb., 247-ft wingspan, 72 control surfaces, differential thrust
- Change in weight distribution
- 40-ft tip deflection
- Divergent pitch oscillations, doubling every 8 seconds
- Airspeed > 2.5 x limit



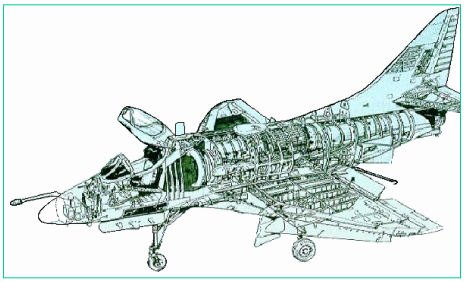
22

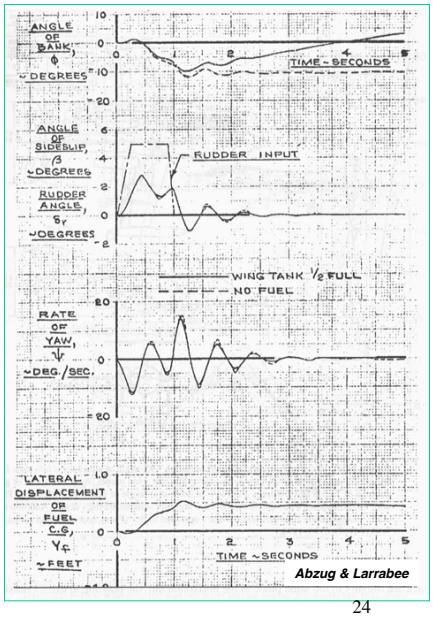
Fuel Shift and Slosh

23

Fuel Shift

- Problem with *partially filled fuel tank*
- Single wing tank from tip to tip (*A4D*)
- Slow, quasi-static shift of fuel c. m.
- Rudder step throws fuel to one side, producing a strong rolling moment

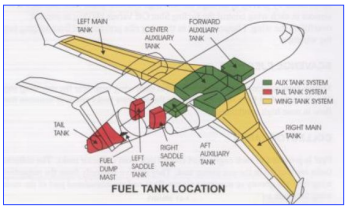


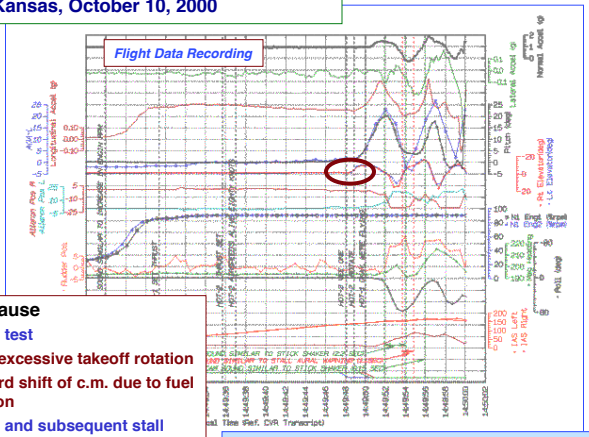


24

Fuel Shift

- NTSB/AAB-04/01
- Loss of Control and Impact with Terrain
- Canadair Challenger CL-604 Flight Test Airplane, C-FTBZ
- Wichita, Kansas, October 10, 2000





Angle of Attack

Normal Acceleration

Pitch Angle

Elevator Angle

IAS

Altitude

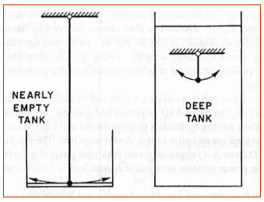
- Probable Cause
  - Aft c.m. test
  - Pilot's excessive takeoff rotation
  - Rearward shift of c.m. due to fuel migration
  - Pitchup and subsequent stall
  - Inadequate test planning

http://www.ntsb.gov/investigations/fulltext/aab0401.html

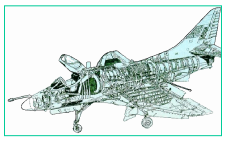
25

Fuel Slosh

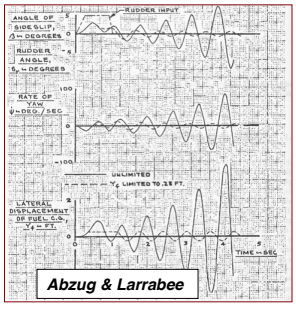
- Dynamic oscillation of fuel center of mass, wave motion at the fuel's surface
- Pendulum and spherical-tank analogies
- Problem is greatest when tank is half-full



- Fuselage tank forward of the aircraft's center of mass *(A4D)*
  - Yawing motion excites oscillatory slosh that couples with *Dutch roll mode*



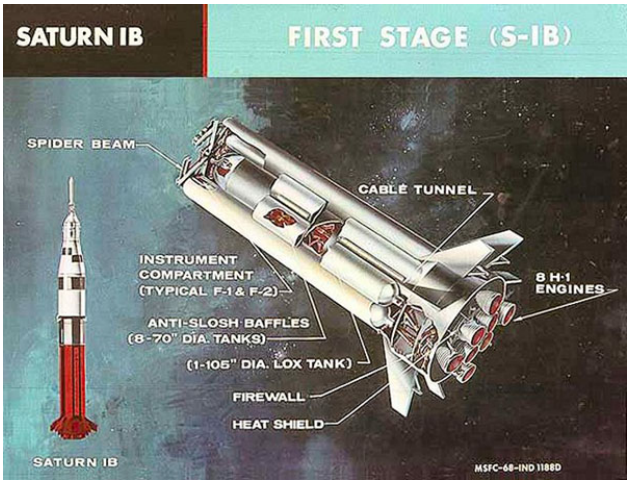
- Fore-aft slosh in wing-tip tanks coupled with the *short period mode (P-80)*



26

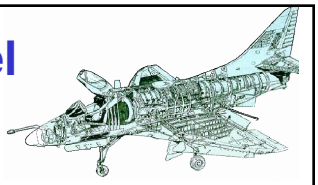
Fuel Slosh

- **Solution: Fuel-tank *baffles***
  - Slow down fuel motion
  - Force resonances to higher frequencies due to smaller cavities
  - Wing internal bracing may act as baffle



27

Problems of Fuel Slosh and Aeroelasticity



- Coupling of non-rigid dynamic modes with rigid-body modes
- Resonant response
  - Dynamically coupled modes of motion with similar frequencies
  - With light damping, oscillatory amplitudes may become large

$$\begin{bmatrix} \Delta\dot{\mathbf{x}}_{aircraft} \\ \Delta\dot{\mathbf{x}}_{elastic} \\ \Delta\dot{\mathbf{x}}_{slosh} \end{bmatrix} = \left[\begin{array}{c|c|c} \mathbf{F}_{aircraft} & \mathbf{F}_{elastic}^{aircraft} & \mathbf{F}_{slosh}^{aircraft} \\ \hline \mathbf{F}_{aircraft}^{elastic} & \mathbf{F}_{elastic} & \mathbf{F}_{slosh}^{elastic} \\ \hline \mathbf{F}_{aircraft}^{slosh} & \mathbf{F}_{elastic}^{slosh} & \mathbf{F}_{slosh} \end{array}\right] \begin{bmatrix} \Delta\mathbf{x}_{aircraft} \\ \Delta\mathbf{x}_{elastic} \\ \Delta\mathbf{x}_{slosh} \end{bmatrix} + \mathbf{G}\Delta\mathbf{u}$$

- Coupling between longitudinal and lateral-directional effects
- Nonlinear aerodynamics
- Exacerbated by floating control surfaces, high hinge moments, and high aerodynamic angles

28

Next Time:
High Speed and Altitude

Flight Dynamics
470-480
Airplane Stability and Control
Chapter 11

Learning Objectives

Effects of air compressibility on flight stability
Variable sweep-angle wings
Aero-mechanical stability augmentation
Altitude/airspeed instability

29