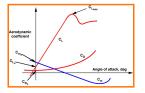
Low-Speed Aerodynamics

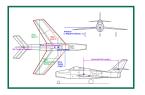
Robert Stengel, Aircraft Flight Dynamics, MAE 331, 2018

Learning Objectives

- · 2D lift and drag
- · Reynolds number effects
- Relationships between airplane shape and aerodynamic characteristics
- · 2D and 3D lift and drag
- Static and dynamic effects of aerodynamic control surfaces







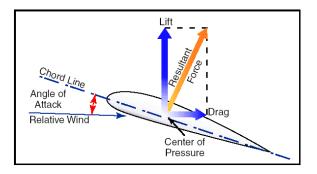
Copyright 2018 by Robert Stengel. All rights reserved. For educational use only. http://www.princeton.edu/~stengel/MAE331.html http://www.princeton.edu/~stengel/FlightDynamics.html

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2-Dimensional Aerodynamic Lift and Drag

Wing Lift and Drag

- · Lift: Perpendicular to free-stream airflow
- Drag: Parallel to the free-stream airflow



3

Longitudinal Aerodynamic Forces

Non-dimensional force coefficients, C_L and C_D , are

dimensionalized by

dynamic pressure, $\frac{1}{q}$, N/m^2 or Ib/sq ft

reference area, S, m² of ft²

$$Lift = C_L \overline{q} S = C_L \left(\frac{1}{2}\rho V^2\right) S$$
$$Drag = C_D \overline{q} S = C_D \left(\frac{1}{2}\rho V^2\right) S$$

$$Drag = C_D \overline{q} S = C_D \left(\frac{1}{2} \rho V^2\right) S$$

Circulation of Incompressible Air Flow About a 2-D Airfoil

Bernoulli's equation (inviscid, incompressible flow) (Motivational, but not the whole story of lift)

$$p_{static} + \frac{1}{2}\rho V^2 = \text{constant along streamline} = p_{stagnation}$$

Vorticity at point x

$$\begin{aligned} & V_{upper}(x) = V_{\infty} + \Delta V(x)/2 \\ & V_{lower}(x) = V_{\infty} - \Delta V(x)/2 \end{aligned}$$

$$\gamma_{2-D}(x) = \frac{\Delta V(x)}{\Delta z(x)}$$

Circulation about airfoil

$$\Gamma_{2-D} = \int_{0}^{c} \gamma_{2-D}(x) dx = \int_{0}^{c} \frac{\Delta V(x)}{\Delta z(x)} dx$$



5

6

Relationship Between Circulation and Lift

Differential pressure along chord section

$$\frac{\Delta p(x)}{} = \left[p_{static} + \frac{1}{2} \rho_{\infty} (V_{\infty} + \Delta V(x)/2)^{2} \right] - \left[p_{static} + \frac{1}{2} \rho_{\infty} (V_{\infty} - \Delta V(x)/2)^{2} \right]$$

$$= \frac{1}{2} \rho_{\infty} \left[(V_{\infty} + \Delta V(x)/2)^{2} - (V_{\infty} - \Delta V(x)/2)^{2} \right]$$

$$= \rho_{\infty} V_{\infty} \Delta V(x) = \rho_{\infty} V_{\infty} \Delta z(x) \gamma_{2-D}(x)$$

2-D Lift (inviscid, incompressible flow)

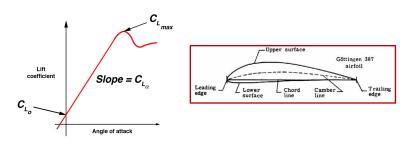
$$\left(Lift\right)_{2-D} = \int_{0}^{c} \Delta p(x) dx = \rho_{\infty} V_{\infty} \int_{0}^{c} \gamma_{2-D}(x) dx = \rho_{\infty} V_{\infty} \left(\Gamma\right)_{2-D}$$

$$\simeq \frac{1}{2} \rho_{\infty} V_{\infty}^{2} c(2\pi\alpha) [\text{thin, symmetric airfoil}] + \rho_{\infty} V_{\infty} (\Gamma_{camber})_{2-D}$$

$$\simeq \frac{1}{2} \rho_{\infty} V_{\infty}^{2} c(C_{L_{\alpha}})_{2-D} \alpha + \rho_{\infty} V_{\infty} (\Gamma_{camber})_{2-D}$$

Lift vs. Angle of Attack

2-D Lift (inviscid, incompressible flow)



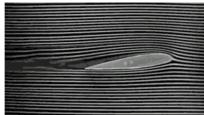
$$(\mathbf{Lift})_{2-D} \simeq \left[\frac{1}{2} \rho_{\infty} V_{\infty}^{2} c \left(C_{L_{\alpha}} \right)_{2-D} \alpha \right] + \left[\rho_{\infty} V_{\infty} \left(\Gamma_{camber} \right)_{2-D} \right]$$

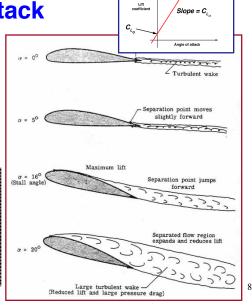
= [Lift due to angle of attack]

+ [Lift due to camber]

Typical Flow Variation with Angle of Attack

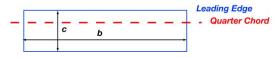
- · At higher angles,
 - flow separates
 - wing loses lift
- Flow separation produces stall





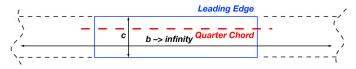
What Do We Mean by 2-Dimensional Aerodynamics?

Finite-span wing -> finite aspect ratio



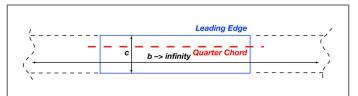
$$AR = \frac{b}{c} \quad rectangular \ wing$$
$$= \frac{b \times b}{c \times b} = \frac{b^2}{S} \quad any \ wing$$

Infinite-span wing -> infinite aspect ratio



9

What Do We Mean by 2-Dimensional Aerodynamics?



Assuming constant chord section, the "2-D Lift" is the same at any y station of the infinite-span wing

$$Lift_{3-D} = C_{L_{3-D}} \frac{1}{2} \rho V^2 S = C_{L_{3-D}} \frac{1}{2} \rho V^2 (bc) \text{ [Rectangular wing]}$$

$$\Delta (Lift_{3-D}) = C_{L_{3-D}} \frac{1}{2} \rho V^2 c \Delta y$$

$$\lim_{\Delta y \to \varepsilon > 0} \Delta \left(Lift_{3-D} \right) = \lim_{\Delta y \to \varepsilon > 0} \left(C_{L_{3-D}} \frac{1}{2} \rho V^2 c \Delta y \right) \Rightarrow \text{"2-D Lift"} \simeq C_{L_{2-D}} \frac{1}{2} \rho V^2 c \Delta y$$

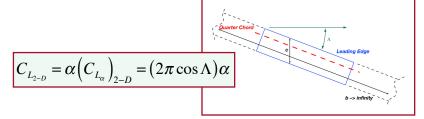
Effect of Sweep Angle on Lift

Unswept wing, symmetric airfoil, 2-D lift slope coefficient Inviscid, incompressible flow

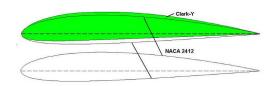
Referenced to chord length, c, rather than wing area

$$C_{L_{2-D}} = \alpha \left(\frac{\partial C_L}{\partial \alpha}\right)_{2-D} = \alpha \left(C_{L_{\alpha}}\right)_{2-D} = (2\pi)\alpha$$
 [Thin Airfoil Theory]

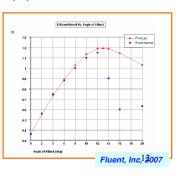
Swept wing, 2-D lift slope coefficient Inviscid, incompressible flow

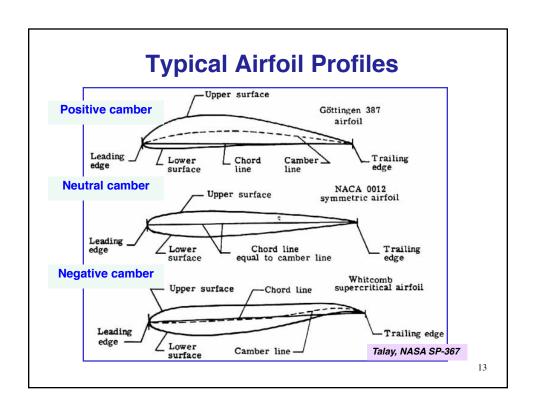


Classic Airfoil Profiles



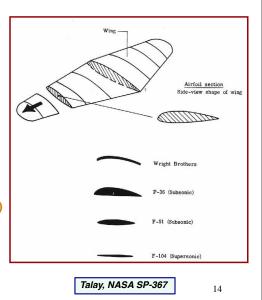
- NACA 4-digit Profiles (e.g., NACA 2412)
 - Maximum camber as percentage of chord (2) = 2%
 - Distance of maximum camber from leading edge, (4) = 40%
 - Maximum thickness as percentage of chord (12) = 12%
- <u>Clark Y (1922)</u>: Flat lower surface, 11.7% thickness
 - GA, WWII aircraft
 - Reasonable L/D
 - Benign theoretical stall characteristics
 - Experimental result is more abrupt

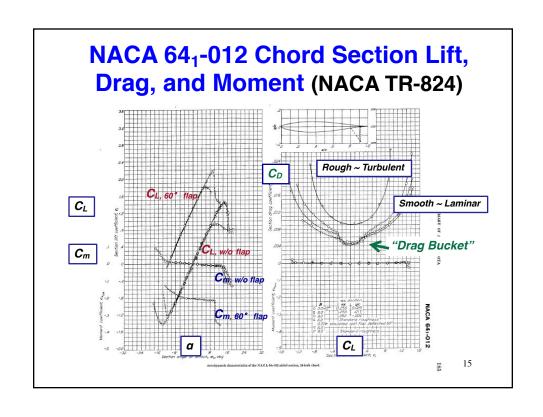


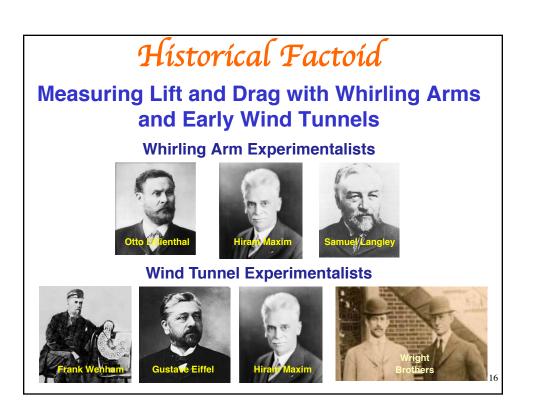


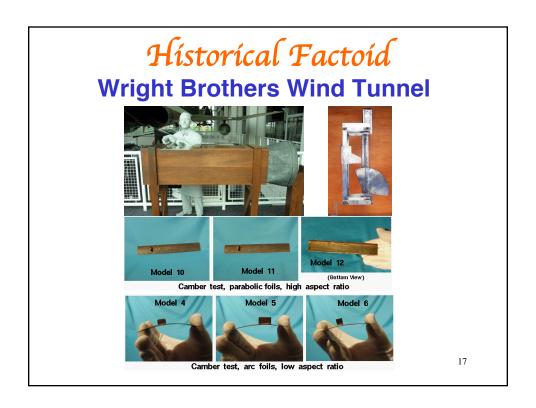


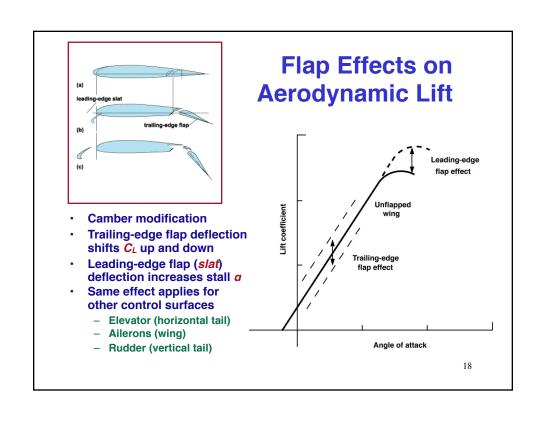
- Camber increases zero-a lift coefficient
- Thickness
 - increases a for stall and softens the stall break
 - reduces subsonic drag
 - increases transonic drag
 - causes abrupt pitching moment variation
- Profile design
 - can reduce center-ofpressure (static margin, TBD) variation with a
 - affects leading-edge and trailing-edge flow separation





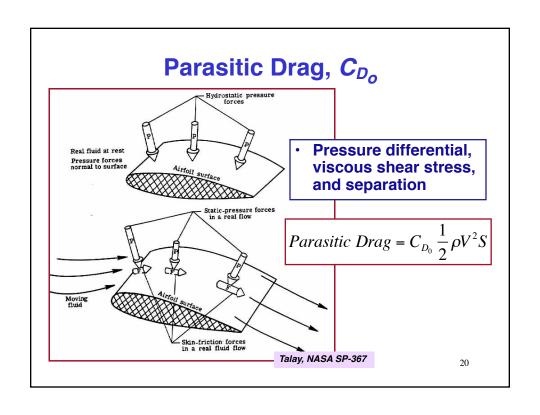






Aerodynamic Drag
$$Drag = C_D \frac{1}{2} \rho V^2 S \approx \left(C_{D_0} + \varepsilon C_L^2\right) \frac{1}{2} \rho V^2 S$$

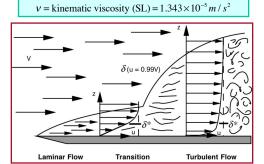
$$\approx \left[C_{D_0} + \varepsilon \left(C_{L_o} + C_{L_a} \alpha\right)^2\right] \frac{1}{2} \rho V^2 S$$
NACA 0012, Re=3 million
$$\frac{0.05}{\sqrt{N} + \sqrt{N} + \sqrt{N}} = \frac{0.04}{\sqrt{N} + \sqrt{N}} = \frac{0.05}{\sqrt{N} + \sqrt{N}} = \frac{0.04}{\sqrt{N} + \sqrt{N}} = \frac{0.05}{\sqrt{N}} = \frac{0.035}{\sqrt{N} + \sqrt{N}} = \frac{0.035}{\sqrt{N}} = \frac{0.035}{\sqrt{N} + \sqrt{N}} = \frac{0.035}{\sqrt{N} + \sqrt{N}} = \frac{0.035}{\sqrt{N}} = \frac{0.035$$



Reynolds Number and Boundary Layer

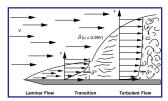
Reynolds Number =
$$\text{Re} = \frac{\rho Vl}{\mu} = \frac{Vl}{v}$$

where $\rho = \text{air density, kg/m}^2$ V = true airspeed, m/s l = characteristic length, m $\mu = \text{absolute (dynamic) viscosity} = 1.725 \times 10^{-5} \, kg \, / \, m \cdot s$



21

Reynolds Number, Skin Friction, and Boundary Layer



Skin friction coefficient for a flat plate

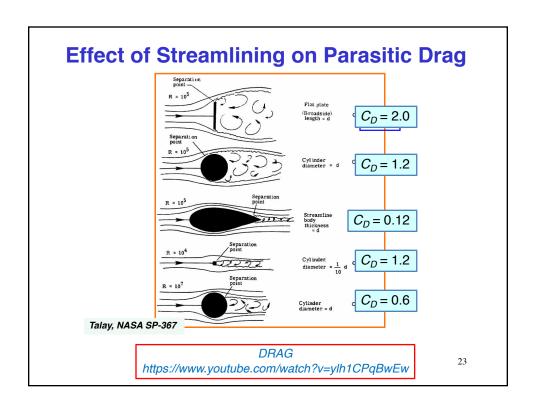
$$C_f = \frac{Friction\ Drag}{\overline{q}S_{wet}}$$
 where $S_{wet} =$ wetted area

Wetted Area: Total surface area of the wing or aircraft, subject to skin friction

Boundary layer thickens in transition, then thins in turbulent flow

$$C_f \approx 1.33 \,\mathrm{Re}^{-1/2} \, \left[laminar \, flow \right]$$

 $\approx 0.46 \left(\log_{10} \mathrm{Re} \right)^{-2.58} \, \left[turbulent \, flow \right]$



Subsonic C_{Do} **Estimate** (Raymer)

Table 12.3 Equivalent skin friction coefficients

$C_{D_0} = C_{fe} rac{S_{ m wet}}{S_{ m ref}}$	C_{fe} -subsonic
Bomber and civil transport	0.0030
Military cargo (high upsweep fuselage)	0.0035
Air Force fighter	0.0035
Navy fighter	0.0040
Clean supersonic cruise aircraft	0.0025
Light aircraft – single engine	0.0055
Light aircraft – twin engine	0.0045
Prop seaplane	0.0065
Jet seaplane	0.0040



Historical Factoid

Wilbur (1867-1912) and **Orville** (1871-1948) Wright





- **Bicycle mechanics from Dayton, OH**
- Self-taught, empirical approach to flight
- Wind-tunnel, kite, and glider experiments
- Dec 17, 1903: Powered, manned aircraft flight ends in success



Historical Factoids

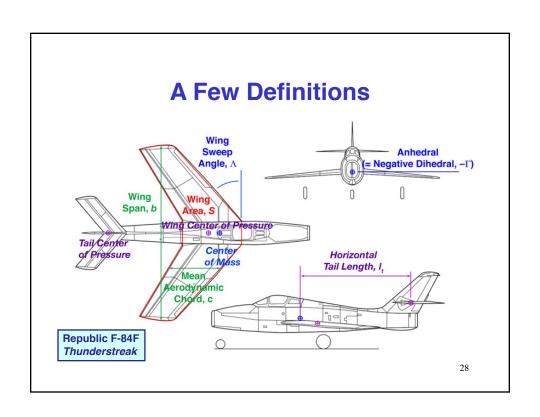
- 1906: 2nd successful aviator: Alberto Santos-Dumont, standing!
 - High dihedral, forward control surface
- Wrights secretive about results until 1908; few further technical contributions
- 1908: Glenn Curtiss et al incorporate
 - Separate aileron surfaces at right
 - Wright brothers sue for infringement of 1906 US patent (and win)
 - 1909: Louis Bleriot's flight across the English Channel

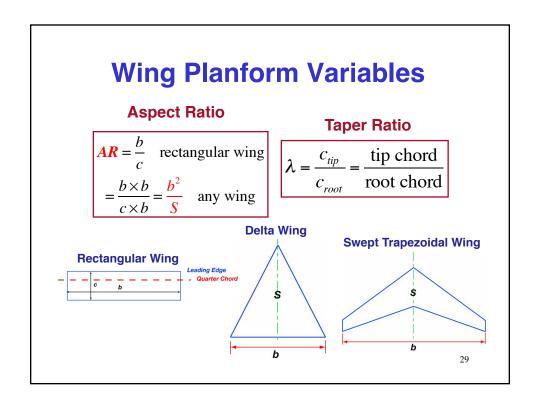


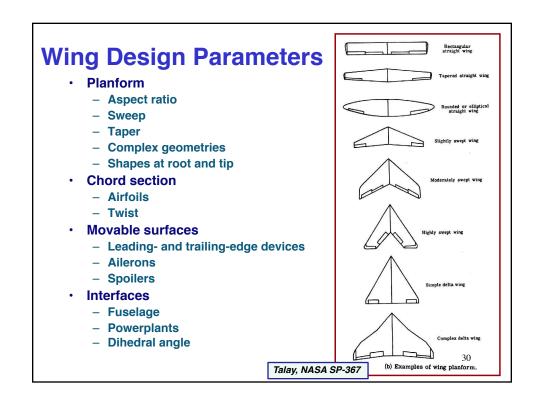


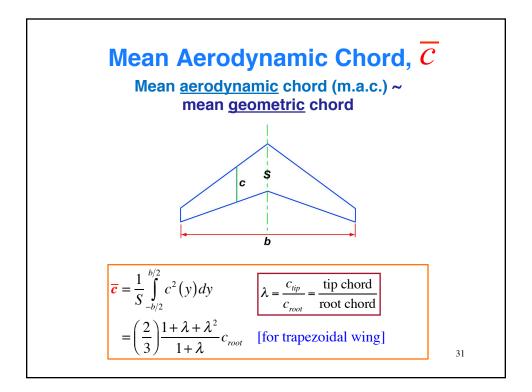


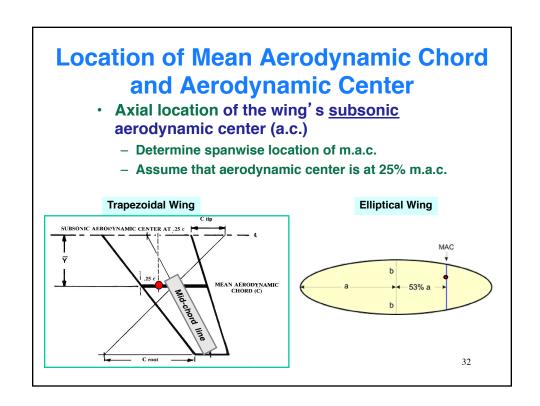
Description of Aircraft Configurations

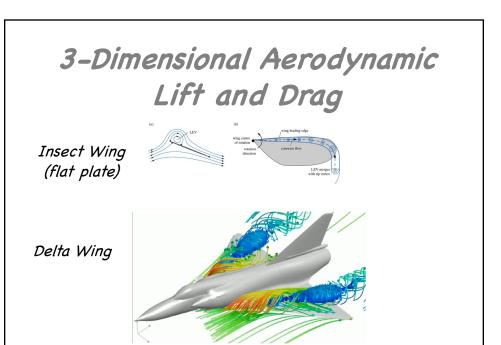








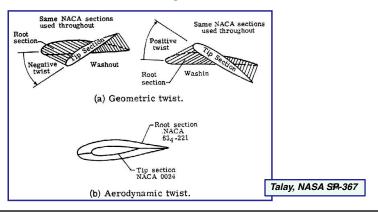




Washout twist

Wing Twist Effects

- reduces tip angle of attack
- typical value: 2° 4°
- changes lift distribution (interplay with taper ratio)
- reduces likelihood of tip stall
- allows stall to begin at the wing root
 - separation"burble" produces buffet at tail surface, warning of stall
- improves aileron effectiveness at high a



Aerodynamic Strip Theory

- · Airfoil section may vary from tip-to-tip
 - Chord length
 - Airfoil thickness
 - Airfoil profile
 - Airfoil twist
- 3-D Wing Lift: Integrate 2-D lift coefficients of airfoil sections across finite span

Incremental lift along span

$$dL = C_{L_{2-D}}(y)c(y)\overline{q}dy$$

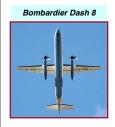
$$= \frac{dC_{L_{3-D}}(y)}{dy}c(y)\overline{q}dy$$

3-D wing lift

$$L_{3-D} = \int_{-b/2}^{b/2} C_{L_{2-D}}(y)c(y)\overline{q} \, dy$$



٥-



Effect of Aspect Ratio on 3-Dimensional Wing Lift Slope Coefficient (Incompressible Flow)



High Aspect Ratio (> 5) Wing

$$C_{L_{\alpha}} \triangleq \left(\frac{\partial C_{L}}{\partial \alpha}\right)_{3-D} = \frac{2\pi AR}{AR+2} = 2\pi \left(\frac{AR}{AR+2}\right)$$

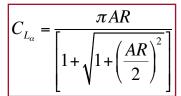
Low Aspect Ratio (< 2) Wing

$$C_{L_{\alpha}} = \frac{\pi AR}{2} = 2\pi \left(\frac{AR}{4}\right)$$

Effect of Aspect Ratio on 3-D Wing Lift Slope Coefficient (Incompressible Flow)

All Aspect Ratios (Helmbold equation)







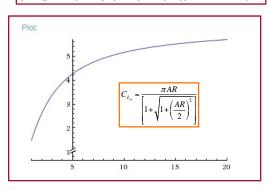
37

Effect of Aspect Ratio on 3-D Wing Lift Slope Coefficient

All Aspect Ratios (Helmbold equation)

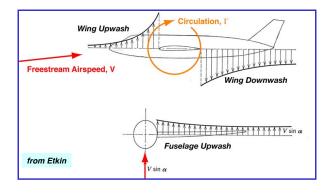
Wolfram Alpha (https://www.wolframalpha.com/)

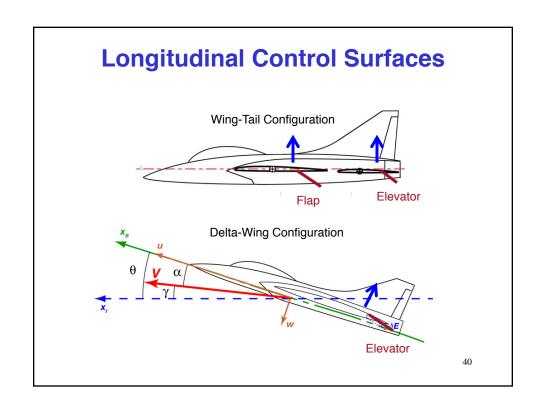
plot(pi A / (1+sqrt(1 + (A / 2)^2)), A=1 to 20)



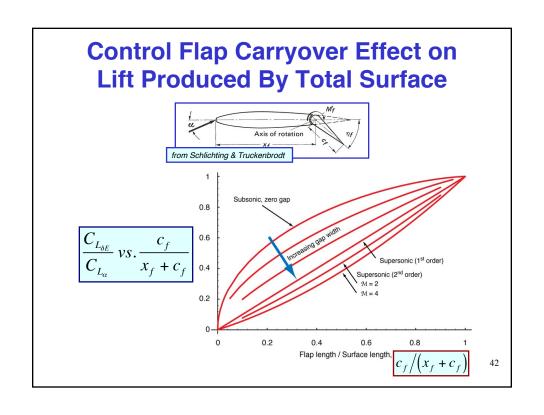
Wing-Fuselage Interference Effects

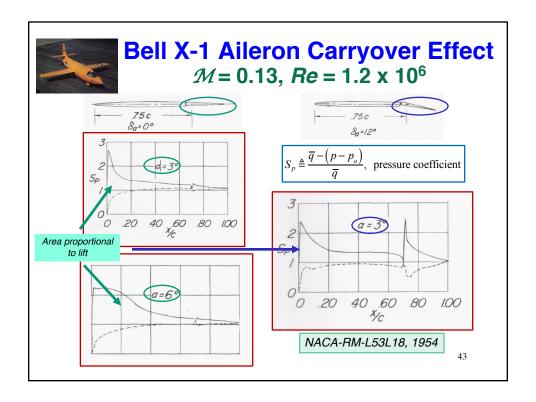
- Wing lift induces
 - Upwash in front of the wing affects canard
 - Downwash behind the wing affects aft tail
 - Local angles of attack modified, affecting net lift and pitching moment
- · Flow around fuselage induces upwash on the wing, canard, and tail





Angle of Attack and Control Surface Deflection - Horizontal tail with elevator control surface - Horizontal tail at positive angle of attack - Horizontal tail with positive elevator deflection - Wing Chord Length - Chord Length - Chord Length - Chord Length





Lift due to Elevator Deflection

Lift coefficient variation due to elevator deflection

$$C_{L_{\delta E}} \triangleq \frac{\partial C_{L}}{\partial \delta E} = \tau_{ht} \eta_{ht} \left(C_{L_{\alpha}} \right)_{ht} \frac{S_{ht}}{S}$$
$$\Delta C_{L} = C_{L_{\delta E}} \delta E$$

 τ_{ht} = Carryover effect

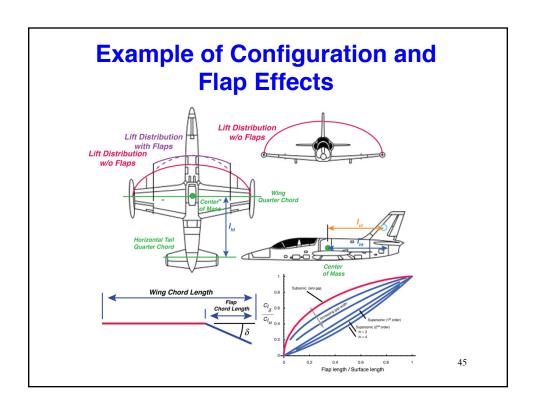
 η_{ht} = Tail efficiency factor

 $\left(C_{L_{\alpha}}\right)_{ht}$ = Horizontal tail lift-coefficient slope

 S_{ht} = Horizontal tail reference area

Lift variation due to elevator deflection

$$\Delta L = C_{L_{\delta E}} \overline{q} S \delta E$$



Next Time: Induced Drag and High-Speed Aerodynamics

Reading:
Flight Dynamics
Aerodynamic Coefficients, 85-96
Airplane Stability and Control
Chapter 1

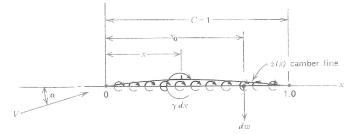
Learning Objectives

Understand drag-due-to-lift and effects of wing planform
Recognize effect of angle of attack on lift and drag coefficients
How to estimate Mach number (i.e., air compressibility) effects on aerodynamics
Be able to use Newtonian approximation to estimate lift and drag

Supplementary Material

47

Thin Airfoil Theory



Downward velocity, w, at x_o due to vortex at x

Differential Integral

$$dw(x_o) = \frac{\gamma(x)dx}{2\pi(x_o - x)}$$

$$w(x_o) = \frac{1}{2\pi} \int_0^1 \frac{\gamma(x)}{(x_o - x)} dx$$

Boundary condition: flow tangent to mean camber line

$$\frac{w(x_o)}{V} = \alpha - \left(\frac{dz}{dx}\right)_{x_o}$$

McCormick, 1995

Thin Airfoil Theory

Integral equation for vorticity

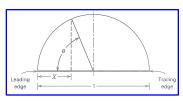
$$\frac{1}{2\pi V} \int_0^1 \frac{\gamma(x)}{(x_o - x)} dx = \alpha - \left(\frac{dz}{dx}\right)_{x_o}$$

Coordinate transformation

$$x = \frac{1}{2} (1 - \cos \theta)$$

Solution for vorticity

$$\gamma = 2V \left[A_0 \frac{1 + \cos \theta}{\sin \theta} + \sum_{n=1}^{\infty} A_n \sin n\theta \right]$$



Coefficients

$$A_0 = \alpha - \frac{1}{\pi} \int_0^{\pi} \frac{dz}{dx} d\theta$$
$$A_n = \frac{2}{\pi} \int_0^{\pi} \frac{dz}{dx} \cos n\theta d\theta$$

McCormick, 1995

49

Thin Airfoil Theory

Lift, from Kutta-Joukowski theorem

$$L = \int_{0}^{1} \rho V \gamma(x) dx = 2\pi A_{0} + \pi A_{1}$$

For thin airfoil with circular arc

$$A_0 = \alpha, \quad A_1 = 4z_{\text{max}}$$

$$C_{L_{2-D}} = 2\pi\alpha + 4\pi z_{\text{max}} = C_{L_{\alpha}}\alpha + C_{L_{o}} \quad \text{[Circular arc]}$$
$$= C_{L_{\alpha}}\alpha \quad \text{[Flat plate]}$$

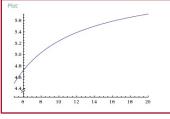
$$C_{L_{\alpha}} = \frac{\partial C_{L}}{\partial \alpha} = 2\pi$$

McCormick, 1995

Effect of Aspect Ratio on 3-Dimensional Wing Lift Slope Coefficient

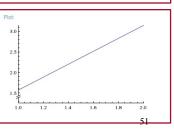
- High Aspect Ratio (> 5) Wing
 - · Wolfram Alpha

plot(2 pi (a/(a+2)), a=5 to 20)



- Low Aspect Ratio (< 2) Wing
 - Wolfram Alpha

plot(2 pi (a / 4), a=1 to 2)



Aerodynamic Stall, Theory and Experiment

- · Flow separation produces stall
- Straight rectangular wing, AR = 5.536, NACA 0015
- Hysteresis for increasing/decreasing a

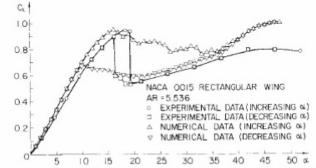
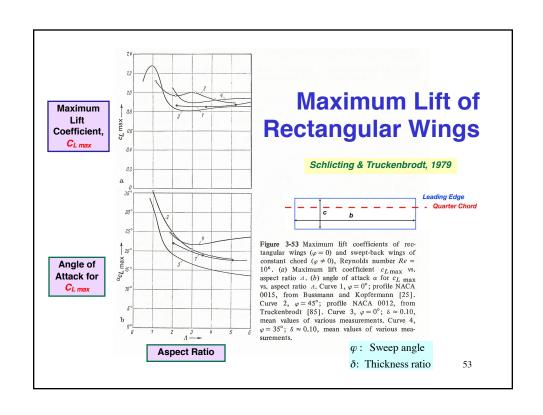
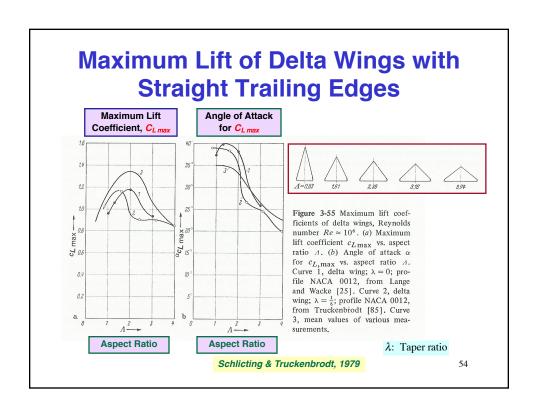
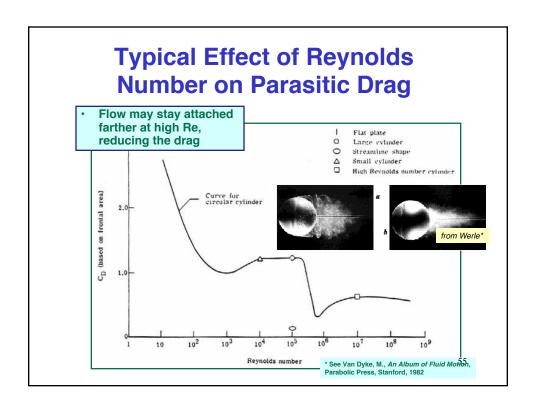


Fig. 3 Lift coefficient vs angle of attack for a rectangular wing with an NACA 0015 airfoil; comparison between experiment 7 and the present numerical technique.

Anderson et al, 1980





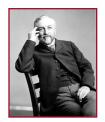




Aft Flap vs. All-Moving Control Surface



- Carryover effect of aft flap
 - Aft-flap deflection can be almost as effective as full surface deflection at subsonic speeds
 - Negligible at supersonic speed
- · Aft flap
 - Mass and inertia lower, reducing likelihood of mechanical instability
 - Aerodynamic hinge moment is lower
 - Can be mounted on structurally rigid main surface



Historical Factoid

Samuel Pierpoint Langley (1834-1906)

- Astronomer supported by Smithsonian Institution
- Whirling-arm experiments
- 1896: Langley's steam-powered Aerodrome model flies 3/4 mile
- Oct 7 & Dec 8, 1903: Manned aircraft flights end in failure







57

Multi-Engine Aircraft of World War II

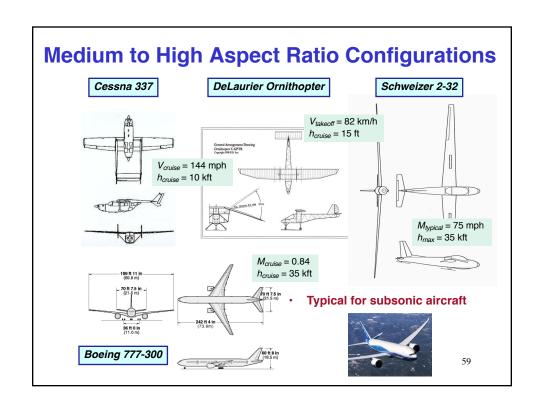


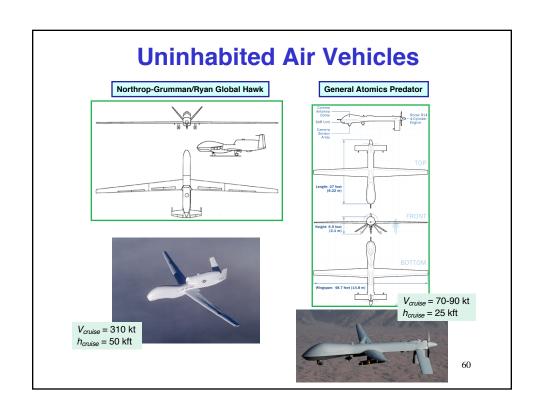


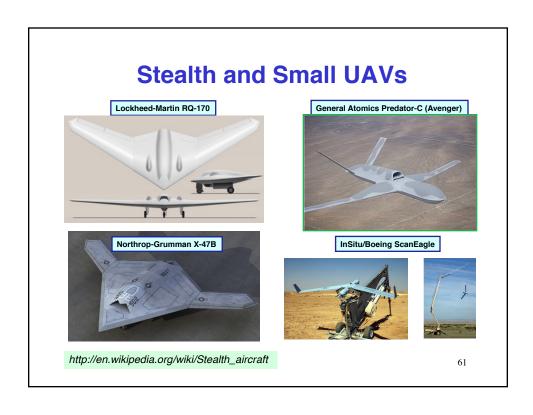


- Large W.W.II aircraft had unpowered controls:
 - High foot-pedal force
 - Rudder stability problems arising from balancing to reduce pedal force
- Severe engine-out problem for twin-engine aircraft



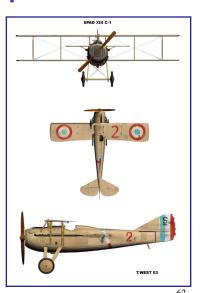






Subsonic Biplane

- Compared to monoplane
 - Structurally stiff (guy wires)
 - Twice the wing area for the same
 - Lower aspect ratio than a single wing with same area and chord
 - Mutual interference
 - Lower maximum lift
 - **Higher drag (interference, wires)**
- Interference effects of two wings
 - Gap
 - **Aspect ratio**
 - Relative areas and spans
 - Stagger



Some Videos

Flow over a narrow airfoil, with downstream vortices

http://www.youtube.com/watch?v=zsO5BQA_CZk

Flow over transverse flat plate, with downstream vortices

http://www.youtube.com/watch?v=0z_hFZx7qvE

Laminar vs. turbulent flow

http://www.youtube.com/watch?v=WG-YCpAGgQQ&feature=related

Smoke flow visualization, wing with flap

http://www.youtube.com/watch?feature=fvwp&NR=1&v=eBBZF_3DLCU/

1930s test in NACA wind tunnel

http://www.youtube.com/watch?v=3_WgkVQWtno&feature=related