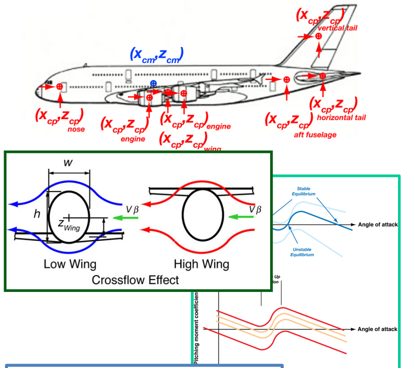
Aerodynamic Moments (i.e., Torques)

Robert Stengel, Aircraft Flight Dynamics, MAE 331, 2018

Learning Objectives

- Aerodynamic balance and moment
- Aerodynamic center, center of pressure, neutral point, and static margin
- Configuration and angle-of-attack effects on pitching moment and stability
- Configuration and sideslip-angle effects on lateral-directional (i.e., rolling and yawing) aerodynamic moments
- Tail design effects on airplane aerodynamics



Reading:
Flight Dynamics
Aerodynamic Coefficients, 96-118

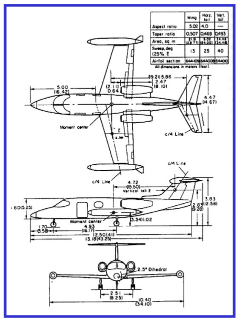
http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

Review Questions

- Why is induced drag proportional to angle of attack squared?
- What spanwise lift distribution gives minimum induced drag?
- Why can lift and drag coefficients be approximated by the Newtonian-flow assumption at very high angle of attack?
- How does profile drag vary with Mach number?
- What are some functions of secondary wing structures?
- What is the primary function of leading edge extensions?
- What is the "Area Rule"?

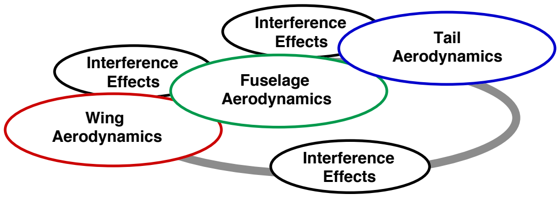
Handbook Approach to Aerodynamic Estimation

- **Build estimates from component effects**
  - **Technical reports, textbooks, ...**
  - **USAF Stability and Control DATCOM (download at http://www.pdas.com/datcomb.html)**
  - **USAF Digital DATCOM (see *Wikipedia* page)**
  - **ESDU Data Sheets (see *Wikipedia* page)**



	Wing	Horz. tail	Vert. tail
Aspect ratio	5.02	4.0	—
Taper ratio	0.507	0.469	0.493
Area, sq m	21.51 [illegible]	5.02 (54.00)	[illegible] (34.48)
Sweep, deg (25% c̄	13	25	40
Airfoil section	64A109	64A008	64A010

All dimensions in meters (feet)



Interference Effects

Tail Aerodynamics

Interference Effects

Fuselage Aerodynamics

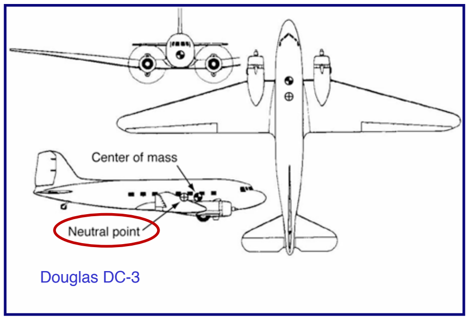
Wing Aerodynamics

Interference Effects

Moments of the Airplane

Airplane Balance

- <u>Conventional aft-tail configuration</u>
  - c.m. near wing's aerodynamic center [point at which wing's pitching moment coefficient is invariant with angle of attack ~25% mean aerodynamic chord (mac)]
- <u>Tailless airplane</u>: c.m. ahead of the <u>neutral point</u>



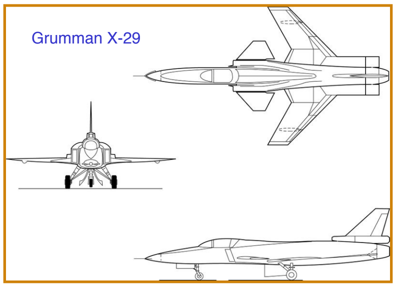
Douglas DC-3

Northrop N-9M

courtsey of Bernardo Malfitano
AirShowFan.com

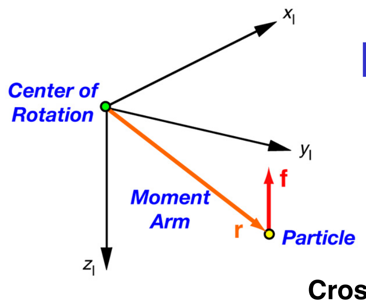
Airplane Balance

- <u>Canard configuration</u>:
  - Neutral point moved forward by canard surfaces
  - Center of mass may be behind the neutral point, requiring closed-loop stabilization
- Fly-by-wire feedback control can expand envelope of allowable center-of-mass locations



Grumman X-29

McDonnell-Douglas X-36



Moment Produced By Force on a Particle

$\tilde{\mathbf{r}}$: Cross-product-equivalent matrix

Cross Product of Vectors

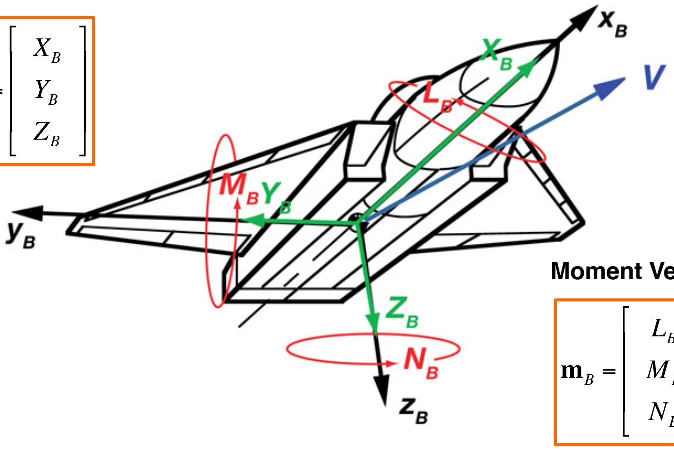
$$\mathbf{r}\times\mathbf{f} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ f_x & f_y & f_z \end{vmatrix} = \left(yf_z - zf_y\right)\boldsymbol{i} + \left(zf_x - xf_z\right)\boldsymbol{j} + \left(xf_y - yf_x\right)\boldsymbol{k}$$

$$\mathbf{m} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \begin{bmatrix} \left(yf_z - zf_y\right) \\ \left(zf_x - xf_z\right) \\ \left(xf_y - yf_x\right) \end{bmatrix} = \mathbf{r}\times\mathbf{f} \triangleq \tilde{\mathbf{r}}\mathbf{f} = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

Forces and Moments Acting on Entire Airplane

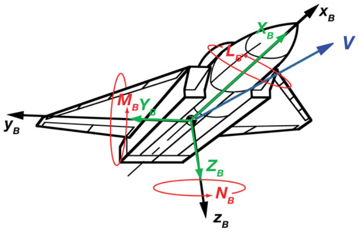
Force Vector

$$\mathbf{f}_B = \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix}$$



Moment Vector

$$\mathbf{m}_B = \begin{bmatrix} L_B \\ M_B \\ N_B \end{bmatrix}$$



Aerodynamic Force and Moment Vectors of the Airplane

Force Vector

$$\mathbf{f}_B = \int_{Surface} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} dx\,dy\,dz = \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix}$$

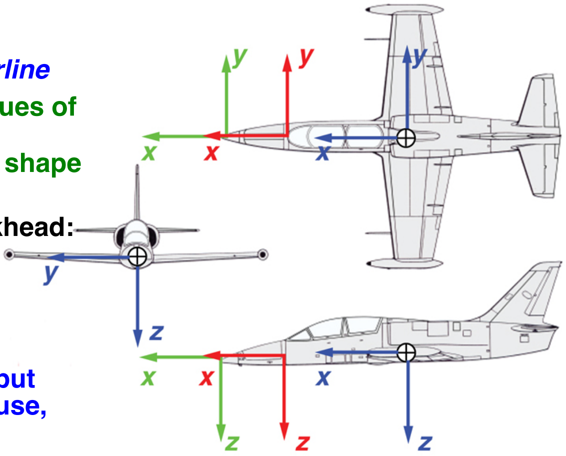
Moment Vector

$$\mathbf{m}_B = \int_{Surface} \begin{bmatrix} \left(yf_z - zf_y\right) \\ \left(zf_x - xf_z\right) \\ \left(xf_y - yf_x\right) \end{bmatrix} dx\,dy\,dz = \begin{bmatrix} L_B \\ M_B \\ N_B \end{bmatrix}$$

Pitching Moment of the Airplane

Body-Axis Reference Frames

- **Reference frame <u>origin</u> is arbitrary; it is a *fiducial point***
  - ***x* axis along *centerline***
  - **Tip of nose: All values of *x* on airframe are negative, but nose shape could change**
  - **Forward-most bulkhead: Fixed for all manufacturing measurements**
  - **Center of mass: Rotational center, but changes with fuel use, payload, etc.**

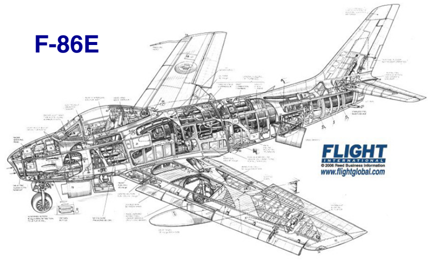


F-86 Nose Variations

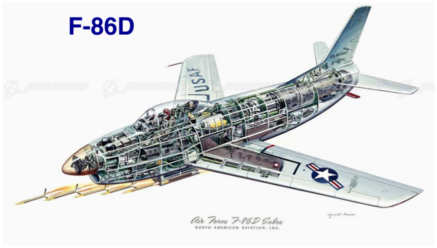
F-86A

F-86D

F-86E



F-86D



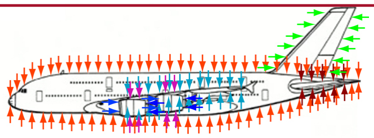
Pitching Moment

(moment about the *y* axis)

Pressure and shear stress differentials x moment arms

Integrate over the airplane <u>surface</u> to produce a net pitching moment

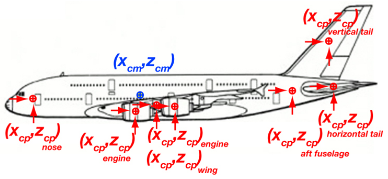
$$\textit{Body - Axis Pitching Moment} = M_B$$
$$= -\iint\limits_{surface} \left[\Delta p_z(x,y) + \Delta s_z(x,y)\right](x - x_{cm})\,dx\,dy$$
$$+ \iint\limits_{surface} \left[\Delta p_x(y,z) + \Delta s_x(y,z)\right]\Delta p_x(z - z_{cm})\,dy\,dz$$



Pitching Moment

(moment about the *y* axis)

Distributed effects can be aggregated to local centers of pressure indexed by *i*



$$M_B \approx -\sum_{i=1}^{I} Z_i(x_i - x_{cm}) + \sum_{i=1}^{I} X_i(z_i - z_{cm})$$
$$+ \textit{Interference Effects} + \textit{Pure Couples}$$

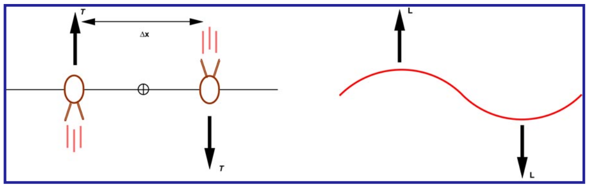
Net effect expressed as

$$\boxed{M_B = C_m \bar{q} S \bar{c}}$$

Pure Couple

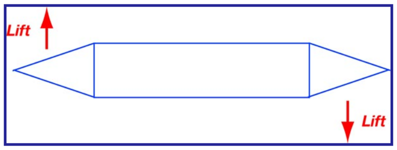
- Net force = 0
- Net moment ≠ 0

Rockets **Cambered Lifting Surface**

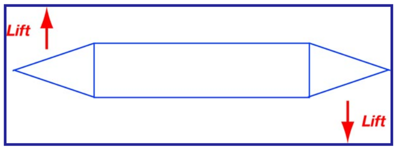


- Cross-sectional area, *S(x)*
- *x* positive to the right
- At small α
  - Positive lift slope with *dS(x)/dx* > 0
  - Negative lift slope with *dS(x)/dx* < 0
- Fuselage typically produces a destabilizing (positive) pitching moment ["Apparent mass" effect]

Fuselage



Lift Coefficient of a Cone



Munk's airship theory (potential flow)

$$C_{L_{cone}} \simeq \pm 2\ fcn\left(\frac{Length}{Diameter}\right)\frac{S_{base}}{S}\alpha$$

$$\simeq \pm 2 \left\{\begin{matrix} 0.4, & Len/Dia = 2 \\ 0.84, & Len/Dia = 5 \\ 0.94, & Len/Dia = 10 \end{matrix}\right\}\frac{S_{base}}{S}\alpha$$

S_{base}: cross-sectional area where flow separates

Munk, NACA-TR-184, 1924

Net Center of Pressure

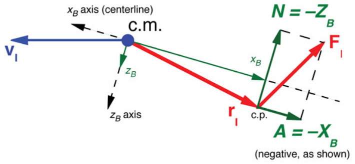
Local centers of pressure can be aggregated at a net center of pressure (or neutral point) along the body *x* axis

$$x_{cp_{net}} = \frac{\left[\left(x_{cp}C_N\right)_{wing} + \left(x_{cp}C_N\right)_{fuselage} + \left(x_{cp}C_N\right)_{tail} + \ldots\right]}{C_{N_{total}}}$$

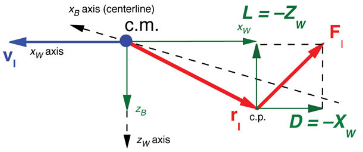
S = reference area

$$C_N = -C_Z$$
$$C_A = -C_X$$

Body Axes



Wind Axes (w.r.t, velocity vector)



Static Margin

- **Static margin *(SM)* reflects the distance between the center of mass *(cm)* and the net center of pressure *(cp)***
  - **Body axes**
  - **Normalized by mean aerodynamic chord**
  - **Does not reflect *z* position of *center of pressure***
- **Positive *SM* if *cp* is behind *cm***

$$\text{Static Margin} \triangleq SM = \frac{100\left(x_{cm} - x_{cp_{net}}\right)_B}{\bar{c}}, \%$$
$$\equiv 100\left(h_{cm} - h_{cp_{net}}\right)\%$$

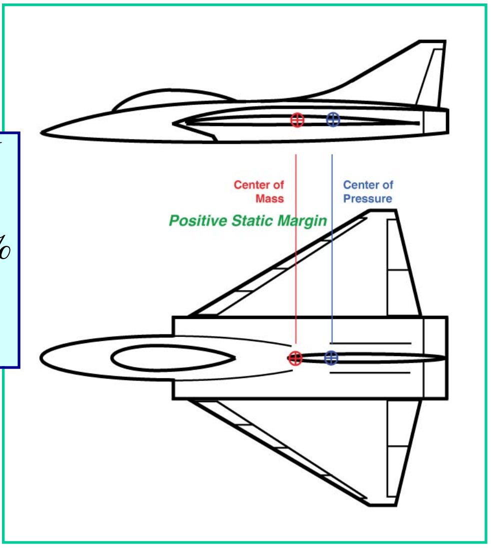
$$h_{cm} \triangleq \frac{x_{cm}}{\bar{c}}$$

Static Margin

$$\text{Static Margin} = SM$$

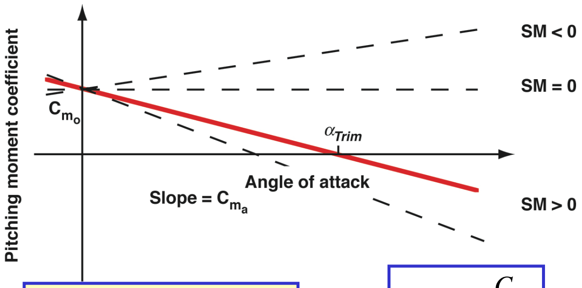
$$= \frac{100\left(x_{cm} - x_{cp_{net}}\right)}{\bar{c}}, \%$$

$$\equiv 100\left(h_{cm} - h_{cp_{net}}\right)\%$$



Effect of Static Margin on Pitching Coefficient

- Zero crossing determines trim angle of attack, i.e., sum of moments = 0
- Negative slope required for static stability
- Slope, $\partial C_m/\partial \alpha$, varies with static margin



$$M_B = \left(C_{m_o} + C_{m_\alpha}\alpha\right)\bar{q}\,S\bar{c}$$

$$\alpha_{Trim} = -\frac{C_{m_o}}{C_{m_\alpha}}$$

Pitch-Moment Coefficient Sensitivity to Angle of Attack

For small angle of attack and no control deflection

$$C_{m_\alpha} \approx -C_{N_{\alpha_{net}}}\left(h_{cm} - h_{cp_{net}}\right) \approx -C_{L_{\alpha_{net}}}\left(h_{cm} - h_{cp_{net}}\right)$$

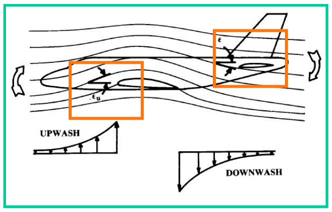
$$\approx -C_{L_{\alpha_{wing}}}\left(\frac{x_{cm} - x_{cp_{wing}}}{\bar{c}}\right) - C_{L_{\alpha_{ht}}}\left(\frac{x_{cm} - x_{cp_{ht}}}{\bar{c}}\right)$$

referenced to wing area, S

Horizontal Tail Lift Sensitivity to Angle of Attack

$$\left[\left(C_{L_{\alpha_{ht}}}\right)_{\substack{horizontal\\ tail}}\right]_{ref=S} = \left(C_{L_{\alpha_{ht}}}\right)_{ref=S_{ht}}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\eta_{elas}\left(\frac{S_{ht}}{S}\right)\left(\frac{V_{ht}}{V_N}\right)^2$$

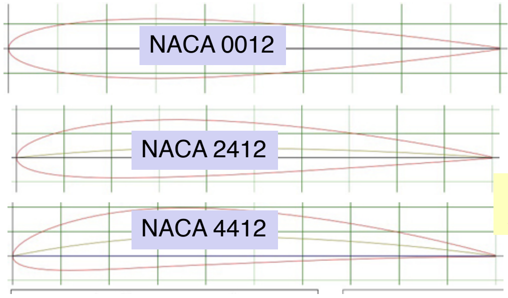
V_{ht} : Airspeed at horizontal tail
ε : **Downwash angle** due to wing at tail
$\partial\varepsilon/\partial\alpha$: **Sensitivity** of downwash to angle of attack
η_{elas} : **Aeroelastic effect**



- Downwash effect on aft horizontal tail
- Upwash effect on a canard (i.e., forward) surface

Aerodynamic Center and Center of Pressure of a Wing

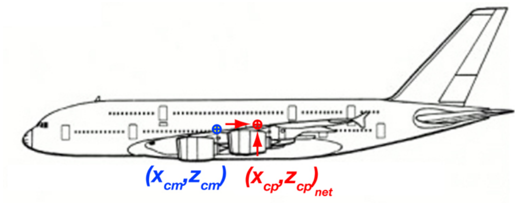
$$\begin{aligned} x_{ac} &= x \text{ for which } \frac{\partial C_m}{\partial \alpha} \equiv 0 \\ &= x_{cp} \text{ for a symmetric airfoil} \\ &\neq x_{cp} \text{ for an asymmetric airfoil} \end{aligned}$$



Airfoil Tools
http://airfoiltools.com

Effect of Static Margin on Pitching Moment

For small angle of attack and no control deflection



$$\begin{aligned} M_B = C_m \bar{q} S \bar{c} &\approx \left[C_{m_o} - C_{N_\alpha} \left(h_{cm} - h_{cp_{net}} \right) \alpha \right] \bar{q} S \bar{c} \\ &\approx \left[C_{m_o} - C_{L_\alpha} \left(h_{cm} - h_{cp_{net}} \right) \alpha \right] \bar{q} S \bar{c} \end{aligned}$$

Effect of Static Margin on Pitching Moment

Sum of moments is zero in *trimmed condition*

$$
\begin{aligned}
M_B &= \left(C_{m_o} + C_{m_\alpha}\alpha\right)\bar{q}\,S\bar{c} \\
&= 0 \quad \text{in trimmed (equilibrium) flight}
\end{aligned}
$$

Typically, static margin is positive and $\partial C_m/\partial\alpha$ is negative for static pitch stability

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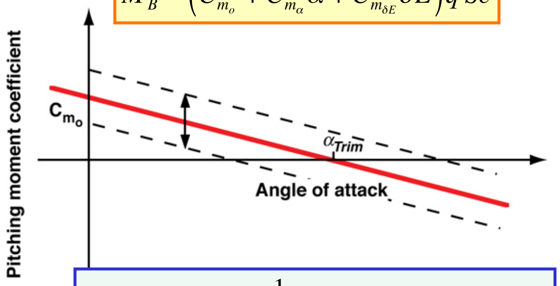
Effect of Elevator Deflection on Pitching Coefficient

Control deflection shifts curve up and down, affecting trim angle of attack

$$
M_B = \left(C_{m_o} + C_{m_\alpha}\alpha + C_{m_{\delta E}}\delta E\right)\bar{q}\,S\bar{c}
$$



$$
\alpha_{Trim} = -\frac{1}{C_{m_\alpha}}\left(C_{m_o} + C_{m_{\delta E}}\delta E\right)
$$

Elevator deflection effectively changes C_{m_o}

Historical Factoids

Aviation in The Great War

- **1914-18:** World War I changes the complexion of flying
  - Reconnaissance
  - Air superiority (dog fights)
  - Bombing
  - Personal transport
- Wrights' US monopoly broken by licensing for war effort
- Aircraft Design
  - Biplanes, a few mono- and triplanes
  - Design for practical functions
  - Multiple engines, larger aircraft
  - Aft tails
  - Increased maneuverability, speed, g-loads, altitude
  - Improved piston engines
  - Tractor propellers

Maneuvering World War I Aircraft

- Maneuverable aircraft with idiosyncrasies
  - Rotary engine
  - Small tail surfaces
  - Reliability issues
- Maneuvering to *stalls* and *spins*
- *Snap roll*: rudder and elevator
- *Barrel roll* : aileron
- *Cross-control* (e.g., right rudder, left stick)
  - glide path control during landing
  - good view of landing point
- Unintended snap rolls led to *spins* and accidents during takeoff or landing

DeHavilland DH-2

http://www..com/watch?v=6ETc1mNNQg8youtube

http://www.youtube.com/watch?v=OBH_Mb0Kj2s

Stability OR Control?

Stability AND Control

- Need for better understanding of Flying (or Handling) Qualities
  - Stability and controllability characteristics as perceived by the pilot
- Desired attributes: Stability of the S.E.-5 and controllability of the D.VII

Lateral-Directional Effects of Sideslip Angle

Rolling and Yawing Moments of the Airplane

Distributed effects can be aggregated to local centers of pressure indexed by *i*

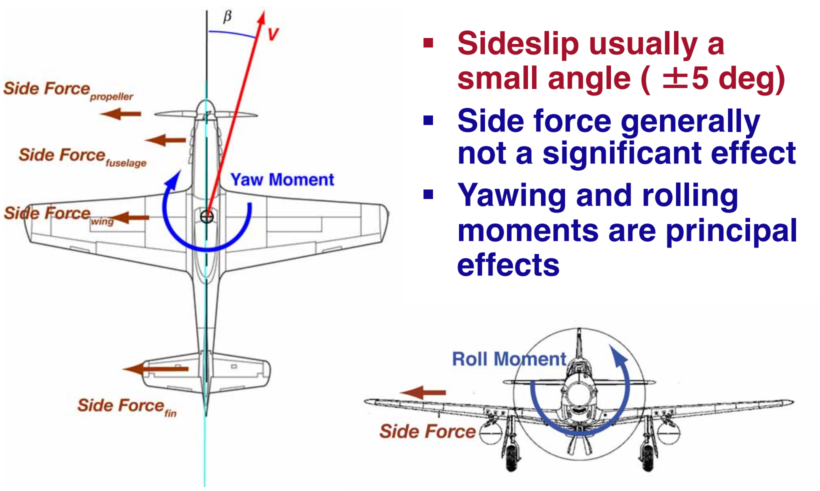
Rolling Moment

$$L_B \approx \sum_{i=1}^{I} Z_i\left(y_i - y_{cm}\right) - \sum_{i=1}^{I} Y_i\left(z_i - z_{cm}\right)$$
$$+ Interference\ Effects + Pure\ Couples$$

Yawing Moment

$$N_B \approx \sum_{i=1}^{I} Y_i\left(x_i - x_{cm}\right) - \sum_{i=1}^{I} X_i\left(y_i - y_{cm}\right)$$
$$+ Interference\ Effects + Pure\ Couples$$

Sideslip Angle Produces Side Force, Yawing Moment, and Rolling Moment



- Sideslip usually a small angle (±5 deg)
- Side force generally not a significant effect
- Yawing and rolling moments are principal effects

Side Force due to Sideslip Angle

$$Y \approx \frac{\partial C_Y}{\partial \beta} \bar{q} S \bullet \beta = C_{Y_\beta} \bar{q} S \bullet \beta$$

- **Fuselage, vertical tail, nacelles, and wing are main contributors**

$$C_{Y_\beta} \approx \left(C_{Y_\beta}\right)_{Fuselage} + \left(C_{Y_\beta}\right)_{Vertical\ Tail} + \left(C_{Y_\beta}\right)_{Nacelles} + \left(C_{Y_\beta}\right)_{Wing}$$

S = reference area

Side Force due to Sideslip Angle

$$\left(C_{Y_\beta}\right)_{Vertical\ Tail} \approx \left(\frac{\partial C_Y}{\partial \beta}\right)_{ref = S_{vt}} \eta_{vt} \left(\frac{S_{vt}}{S}\right)$$

$$\left(C_{Y_\beta}\right)_{Fuselage} \approx -2\frac{S_{Base}}{S}; \quad S_B = \pi d_{Base}^2 \Big/ 4$$

$$\left(C_{Y_\beta}\right)_{Wing} \approx -C_{D_{Parasite,Wing}} - k\Gamma^2$$

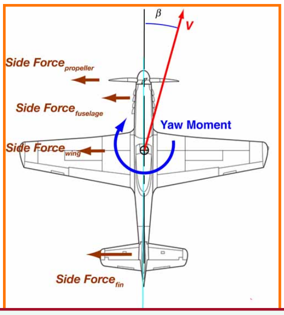
$\eta_{vt} =$ Vertical tail efficiency (p. 96, *Flight Dynamics*)

$$k = \frac{\pi AR}{1 + \sqrt{1 + AR^2}}$$

$\Gamma =$ Wing dihedral angle, rad

Yawing Moment due to Sideslip Angle

$$N \approx \frac{\partial C_n}{\partial \beta}\left(\frac{\rho V^2}{2}\right) Sb \bullet \beta = C_{n_\beta} \bar{q} Sb \bullet \beta$$



- **Side force contributions times respective moment arms**
  - **Non-dimensional stability derivative**

$$C_{n_\beta} \approx \left(C_{n_\beta}\right)_{Vertical\ Tail} + \left(C_{n_\beta}\right)_{Fuselage} + \left(C_{n_\beta}\right)_{Wing} + \left(C_{n_\beta}\right)_{Propeller}$$

S = reference area

Yawing Moment due to Sideslip Angle

Vertical tail contribution

$$\left(C_{n_\beta}\right)_{Vertical\ Tail} \approx -C_{Y_{\beta_{vt}}} \eta_{vt} \frac{S_{vt} l_{vt}}{Sb} \triangleq -C_{Y_{\beta_{vt}}} \eta_{vt} \mathbb{V}_{VT}$$

$l_{vt} \triangleq$ **Vertical tail length** (+)

= distance from center of mass to tail center of pressure

$= x_{cm} - x_{cp_{vt}}$ [x is positive forward; both are negative numbers]

$$\eta_{vt} = \eta_{elas}\left(1 + \partial\sigma / \partial\beta\right)\left(\frac{V_{vt}^2}{V_N^2}\right)$$

$$\mathbb{V}_{VT} = \frac{S_{vt} l_{vt}}{Sb} = \textbf{Vertical Tail Volume Ratio}$$

Yawing Moment due to Sideslip Angle

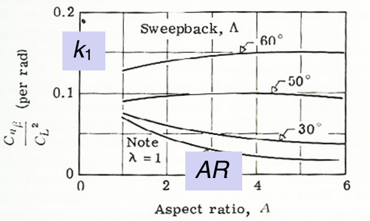
Fuselage contribution

$$\boxed{\left(C_{n_\beta}\right)_{Fuselage} = \frac{-2K\,Volume_{Fuselage}}{Sb}}$$

$$\boxed{K = \left(1 - {d_{max}}\big/{Length_{fuselage}}\right)^{1.3}}$$

Wing (differential lift and induced drag) contribution

$$\boxed{\begin{aligned}\left(C_{n_\beta}\right)_{Wing} &= 0.075 C_{L_N}\Gamma + fcn(\Lambda, AR, \lambda) C_{L_N}{}^2 \\ &\triangleq k_0 C_{L_N}\Gamma + k_1 C_{L_N}{}^2 \quad (\text{eq. } 2.4\text{-}66, \textit{Flight Dynamics})\end{aligned}}$$



Seckel, from NACA TR-1098, 1950

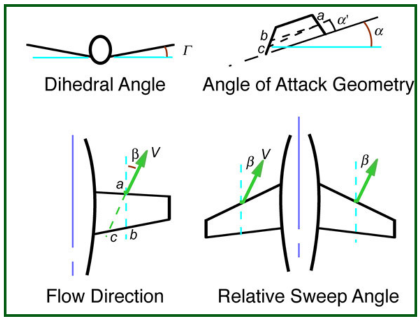
Rolling Moment due to Sideslip Angle

Dihedral effect

$$\boxed{L \approx C_{l_\beta}\bar{q}Sb \bullet \beta}$$

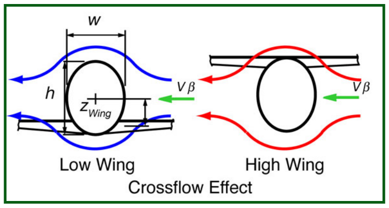
$$\boxed{C_{l_\beta} \approx \left(C_{l_\beta}\right)_{Wing} + \left(C_{l_\beta}\right)_{Wing-Fuselage} + \left(C_{l_\beta}\right)_{Vertical\ Tail}}$$

Unequal lift on left and right wings induces rolling motion



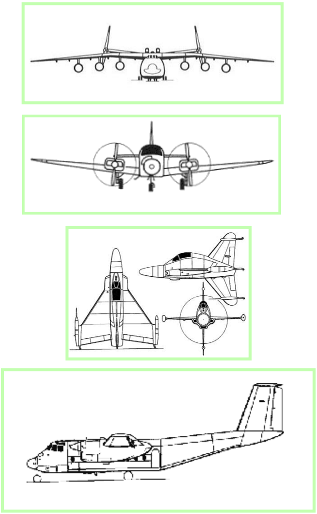
Rolling Moment due to Sideslip Angle

- **Wing vertical location effect: Crossflow produces up- and down-wash**
  - **Rolling effect depends on vertical location of the wing**



Crossflow Effect

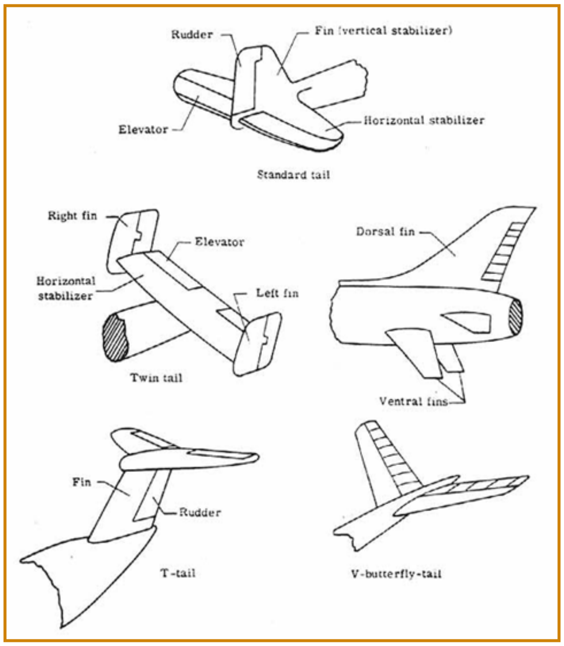
- **Vertical tail effect**



Tail Design Effects

Tail Design Effects

- **Aerodynamics analogous to those of the wing**
- **Longitudinal stability**
  - Horizontal stabilizer
  - Short period natural frequency and damping
- **Directional stability**
  - Vertical stabilizer (fin)
    - Ventral fins
    - Strakes
    - Leading-edge extensions
    - Multiple surfaces
    - Butterfly (V) tail
  - Dutch roll natural frequency and damping
- **Stall or spin prevention/ recovery**
- **Avoid *rudder lock (TBD)***



Horizontal Tail Size and Location


Grumman XF5F


North American F-86

- 15-30% of wing area
- ~ wing semi-span behind the c.m.
- **Must trim neutrally stable airplane at maximum lift in ground effect**
- **Effect on short period mode**
- **Horizontal Tail Volume: Typical value = 0.48**

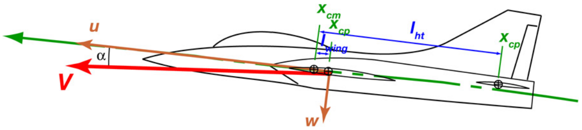
$$\mathbb{V}_{HT} = \frac{S_{ht}}{S}\frac{l_{ht}}{\bar{c}}$$

Tail Moment Sensitivity to Angle of Attack

$$C_{m_{\alpha_{ht}}} = -\left(C_{L_{\alpha_{ht}}}\right)_{ht}\left(\frac{V_{ht}}{V_N}\right)^2\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)\eta_{elas}\left[\left(\frac{S_{ht}}{S}\right)\left(\frac{l_{ht}}{\bar{c}}\right)\right]$$

$$= -\left(C_{L_{\alpha_{ht}}}\right)_{ht}\left(\frac{V_{ht}}{V_N}\right)^2\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)\eta_{elas}\mathbb{V}_{HT}$$

$$\mathbb{V}_{HT} = \frac{S_{ht}l_{ht}}{S\bar{c}} = \textbf{Horizontal Tail Volume Ratio}$$



Pitching Moment due to Elevator Deflection

Normal force coefficient variation due to elevator deflection

$$C_{L_{\delta E}} \triangleq \frac{\partial C_L}{\partial \delta E} = \tau_{ht}\eta_{ht}\left(C_{L_\alpha}\right)_{ht}\frac{S_{ht}}{S} \approx C_{N_{\delta E}}$$

$$\Delta C_N = C_{N_{\delta E}}\delta E$$

$\tau_{ht} =$ Carryover effect

$\eta_{ht} =$ Tail efficiency factor

$\left(C_{L_\alpha}\right)_{ht} =$ Horizontal tail lift-coefficient slope

$S_{ht} =$ Horizontal tail reference area

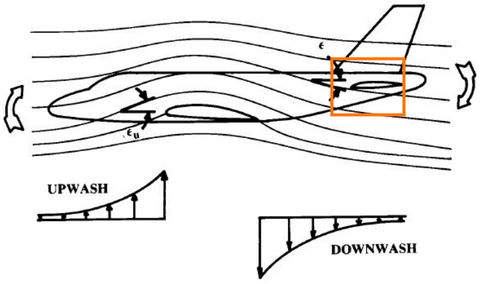
Pitching moment coefficient variation due to elevator deflection

$$C_{m_{\delta E}} = C_{N_{\delta E}}\frac{l_{ht}}{\bar{c}} \approx -\tau_{ht}\eta_{ht}\left(C_{L_\alpha}\right)_{ht}\left(\frac{S_{ht}}{S}\frac{l_{ht}}{\bar{c}}\right)$$

$$= -\tau_{ht}\eta_{ht}\left(C_{L_\alpha}\right)_{ht}\mathbb{V}_{HT}$$

Downwash and Elasticity Also Effect Elevator Sensitivity

$$\left[\left(\frac{\partial C_L}{\partial \delta E}\right)_{ht}\right]_{ref=S} = \left(C_{L_{\delta E}}\right)_{ref=S} = \left(C_{L_{\delta E}}\right)_{ref=S_{ht}} \left(\frac{V_{tail}}{V_N}\right)^2 \left(1-\frac{\partial \varepsilon}{\partial \alpha}\right)\eta_{elas}\left(\frac{S_{ht}}{S}\right)$$



Vertical Tail Location and Size

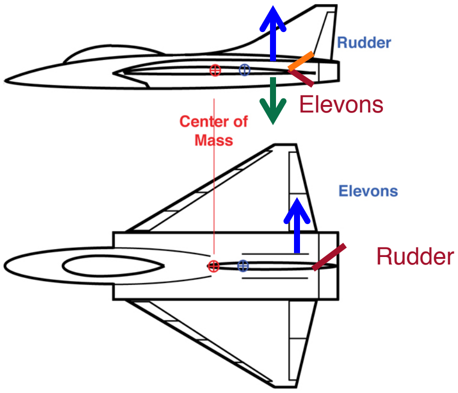
North American P-51B

- Analogous to horizontal tail volume
- Effect on Dutch roll mode
- Powerful rudder for spin recovery
  - Full-length rudder located behind the elevator
  - High horizontal tail so as not to block the flow over the rudder
- Vertical Tail Volume: Typical value = 0.18

$$\mathbb{V}_{VT} = \frac{S_{vt}}{S}\frac{l_{vt}}{b}$$

Otto Koppen: “If they build more than one of these, they’re crazy!”
http://en.wikipedia.org/wiki/Otto_C._Koppen

Lateral-Directional Control Surfaces



Yawing Moment due to Rudder Deflection

Side force coefficient variation due to rudder deflection

$$\left(C_{Y_{\delta R}}\right)_{ref=S} \triangleq \left(\frac{\partial C_Y}{\partial \delta R}\right)_{ref=S} = \left[\left(C_{L_\alpha}\right)_{vt}\right]_{ref=S_{vt}} \tau_{vt}\eta_{vt}\frac{S_{vt}}{S}$$

$$\Delta C_Y = C_{Y_{\delta R}}\delta R$$

Yawing moment coefficient variation due to rudder deflection

$$\left(C_{n_{\delta R}}\right)_{ref=S} = -\left(C_{Y_{\delta R}}\right)_{ref=S}\frac{l_{vt}}{b} \approx -\left[\left(C_{L_\alpha}\right)_{vt}\right]_{ref=S_{vt}} \tau_{vt}\eta_{vt}\left(\frac{S_{vt}}{S}\frac{l_{vt}}{b}\right)$$

$$= -\tau_{vt}\eta_{vt}\left[\left(C_{L_\alpha}\right)_{vt}\right]_{ref=S_{vt}} \mathbb{V}_{VT}$$

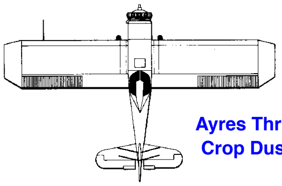
Rolling Moment due to Aileron Deflection

$$\boxed{L \approx C_{l_{L\delta A}} \overline{q} S b \bullet \delta A}$$

For a trapezoidal planform, subsonic flow

$$\left(C_{l_{\delta A}}\right)_{3D} \simeq \left(\frac{C_{L_\delta}}{C_{L_\alpha}}\right)_{2D} \frac{\left(C_{L_\alpha}\right)_{3D}}{1+\lambda}\left[\frac{1-k^2}{3} - \frac{1-k^3}{3}(1-\lambda)\right]$$

$$k \triangleq \frac{y}{b/2}, \quad y = \text{ Inner edge of aileron}, \lambda = \text{ Taper ratio}$$



Ayres Thrush
Crop Duster

Next Time:
Aircraft Performance

Reading:
Flight Dynamics
Aerodynamic Coefficients, 118-130

Learning Objectives

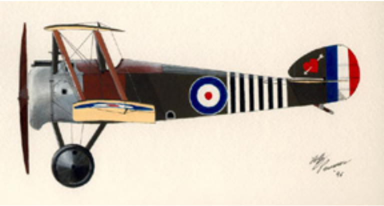
Definitions of airspeed
Performance parameters
Steady cruising flight conditions
Breguet range equations
Optimize cruising flight for minimum thrust and power
Flight envelope

Supplemental Material

Sopwith Camel

SOPWITH F.1 "CAMEL"

Nicknamed the Camel because of the humped fairing over its twin machine guns, the tight-turning Sopwith F.1 performed best at about 12,000 feet. This one was flown by Captain A. Roy Brown, often credited as the conqueror of Richthofen.



- Rotary engine induced gyroscopic coupling
- Highly maneuverable
- Aft fuel tank; when full, center of mass was too far aft for stability
- Vertical tail too small, spin recovery not automatic with centering of controls

http://www.youtube.com/watch?v=3ApowyEXSXM

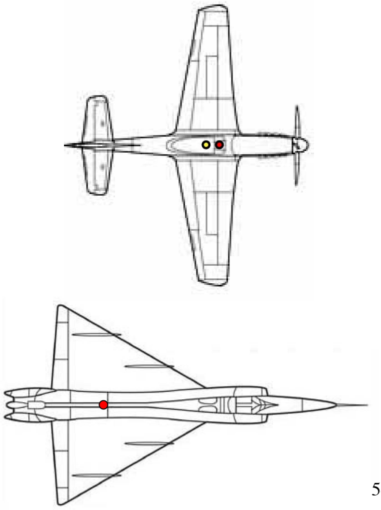
S.E.-5 vs. Fokker D.VII

- ***RAF S.E.-5***: theoretical approach to design
  - "Best WWI design from the Royal Aircraft Factory"
  - Stationary engine
  - High dihedral
  - Stable spiral mode
  - High control forces
  - Poor maneuverability
  - Relatively safe and effective

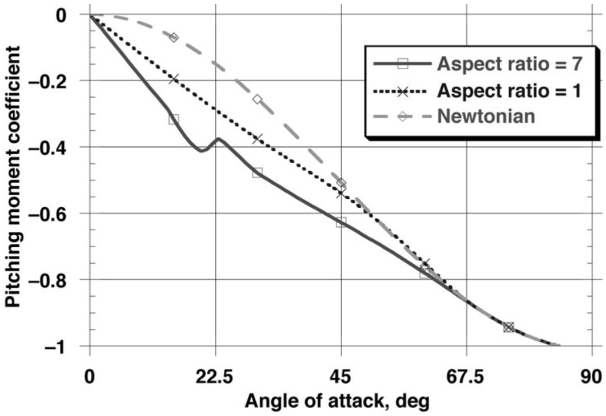
- **Fokker D.VII**: empirical approach to design
  - Horn balances to reduce control forces
  - Stationary engine
  - Neutral-to-negative stability
  - Good maneuverability
  - Relatively dangerous

Planform Effect on Center of Pressure Variation with Mach Number

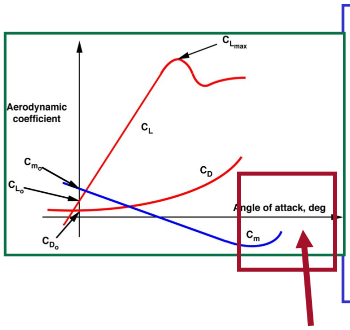
- **Straight Wing**
  - Subsonic center of pressure (c.p.) at ~1/4 mean aerodynamic chord (m.a.c.)
  - Transonic-supersonic c.p. at ~1/2 m.a.c.
- **Delta Wing**
  - Subsonic-supersonic c.p. at ~2/3 m.a.c.



Subsonic Pitching Coefficient vs. Angle of Attack (0° < α < 90°)



"Pitch Up" and Deep Stall



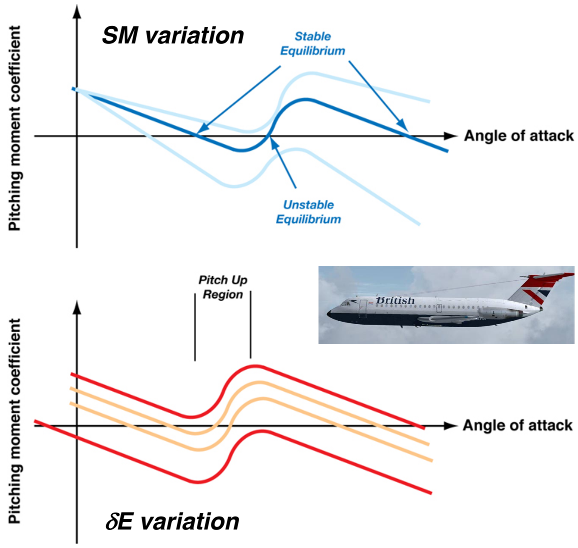
- Possibility of 2 stable equilibrium (trim) points with same control setting
  - Low α
  - High α
- High-angle trim is called **deep stall**
  - Low lift
  - High drag
- Large control moment required to regain low-angle trim

TU-154 Pitch Up Accident
http://www.youtube.com/watch?v=IpZ8YukAwwI&feature=related

BAC 1-11 Deep Stall Flight Testing Accident
http://en.wikipedia.org/wiki/BAC_One-Eleven

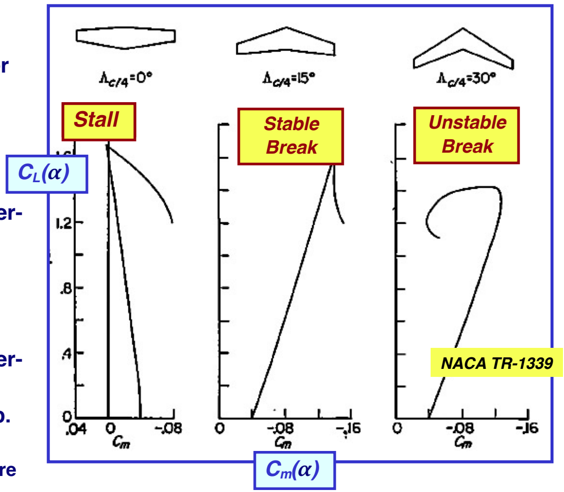
Pitch Up and Deep Stall, C_m vs. α

- Possibility of 2 stable equilibrium (trim) points with same control setting
  - Low α
  - High α
- High-angle trim is called deep stall
  - Low lift
  - High drag
- Large control moment required to regain low-angle trim



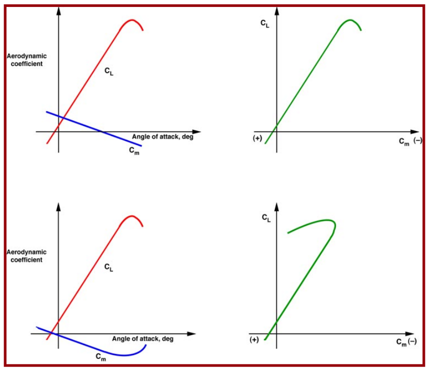
Sweep Effect on Pitch Moment Coefficient, C_L vs. C_m

- $\Lambda_{c/4} = 0$
  - Low α center of pressure (c.p.) in front of the quarter chord
  - Stable break at stall (c.p. moves aft)
- $\Lambda_{c/4} = 15°$
  - Low α c.p. aft of the quarter-chord
  - Stable break at stall (c.p. moves aft)
- $\Lambda_{c/4} = 30°$
  - Low α c.p. aft of the quarter-chord
  - Unstable break at stall (c.p. moves forward)
    - Outboard wing stalls before inboard wing ("tip stall")

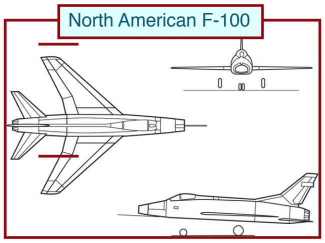


Pitch Up: Explanation of C_L vs. C_m Cross-plot

- Crossplot C_L vs. C_m to obtain plots such as those shown on previous slide
- Positive break in C_m is due to forward movement of net center of pressure, decreasing static margin



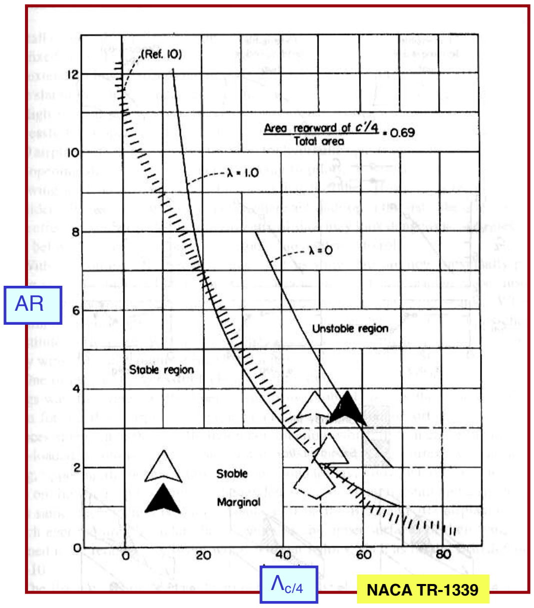
North American F-100



https://www.youtube.com/watch?v=Q2qqKwndFW0

Shortal-Maggin Longitudinal Stability Boundary for Swept Wings

- Stable or unstable pitch break at the stall
- Stability boundary is expressed as a function of
  - Aspect ratio
  - Sweep angle of the quarter chord
  - Taper ratio



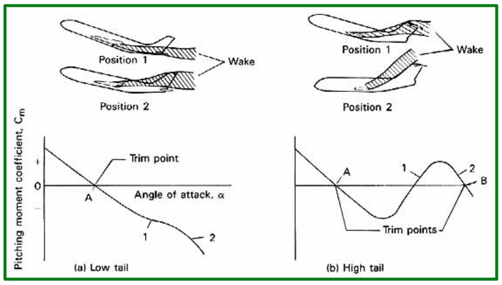
AR

$\Lambda_{c/4}$

NACA TR-1339

Horizontal Tail Location

- Horizontal tail and elevator in wing wake at selected angles of attack
- Effectiveness of high-mounted elevator is unaffected by wing wake at low to moderate angle of attack
- Effectiveness of low tail is unaffected by wing wake at high angle of attack



Twin and Triple Vertical Tails

- Increased tail area with no increase in vertical height
- End-plate effect for horizontal tail improves effectiveness
- Proximity to propeller slipstream

Consolidated B-24

North American B-25

Lockheed C-69

Fairchild-Republic A-10

Ventral Fin Effects

Increase directional stability
Counter roll due to sideslip of the dorsal fin


LTV F8U-3


North American X-15


Learjet 60


Beechcraft 1900D

V (Butterfly) Tails

- Analogous to conventional tail at low angles of attack and sideslip
- Control surface deflection
  - Sum: Pitch control
  - Difference: Yaw control
- Nonlinear effects at high angle of attack are quite different from conventional tail

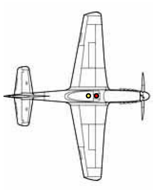

Beechcraft Bonanza

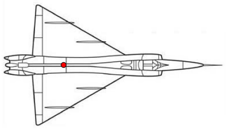

Scaled Composites V-Jet II


Fouga Magister

Effects of Wing Aspect Ratio and Sweep Angle

- Lift slope
- Pitching moment slope
- Lift-to-drag ratio
- All contribute to
  - Phugoid damping
  - Short period natural frequency and damping
  - Roll damping





Effects of Wing Aspect Ratio

- Neglecting air compressibility
- Angles of attack below stall

Lift slope

$$C_{L_{\alpha_{wing}}} = \frac{\pi AR}{\left[1+\sqrt{1+\left(\frac{AR}{2}\right)^2}\right]}$$

Pitching moment slope

$$C_{m_\alpha} \approx -C_{L_{\alpha_{total}}}\left(\frac{\text{Static Margin (\%)}}{100}\right)$$

Lift-to-drag ratio

$$L/D = \frac{C_{L_{total}}}{\left(C_{D_o}+\varepsilon C_L^2\right)_{total}} = \frac{\left(C_{L_o}+C_{L_\alpha}\alpha\right)_{total}}{\left[C_{D_o}+\varepsilon C_L^2\right]_{total}}$$

Roll damping

Wing with taper

$$\left(C_{l_{\hat{p}}}\right)_{wing} = \frac{\partial\left(\Delta C_l\right)_{wing}}{\partial \hat{p}} = -\frac{C_{L_{\alpha_{wing}}}}{12}\left(\frac{1+3\lambda}{1+\lambda}\right)$$

Thin triangular wing

$$\left(C_{l_{\hat{p}}}\right)_{Wing} = -\frac{\pi AR}{32}$$

Propeller Effects

- *Slipstream* over wing, tail, and fuselage
  - Increased dynamic pressure
  - Swirl of flow
  - Downwash and sidewash at the tail
- *DH-2* unstable with engine out
- Single- and multi-engine effects
- Design factors: fin offset (correct at one airspeed only), *c.m.* offset
- *Propeller fin effect*: Visualize lateral/horizontal projections of the propeller as forward surfaces
- Contra-rotating propellers minimize torque and swirl

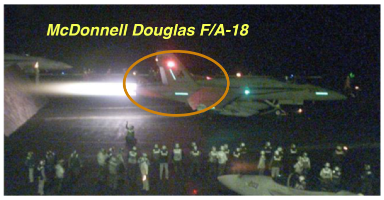

DeHavilland DH-2

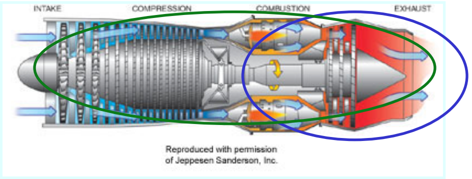

DeHavilland DHC-6


Westland Wyvern

Jet Effects on Rigid-Body Motion


North American F-86


McDonnell Douglas F/A-18

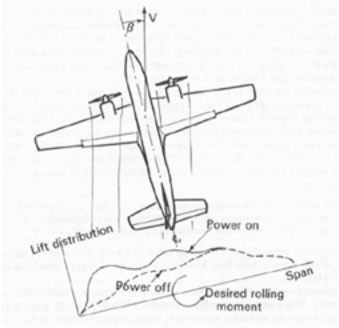


Reproduced with permission of Jeppesen Sanderson, Inc.

- Normal force at intake (analogous to propeller fin effect) (*F-86*)
- Deflection of airflow past tail due to entrainment in exhaust (*F/A-18*)
- Pitch and yaw damping due to internal exhaust flow
- Angular momentum of rotating machinery

Loss of Engine

- **Loss of engine produces large yawing (and sometimes rolling) moment(s), requiring major application of controls**
- **Engine-out training can be as hazardous, especially during takeoff, for both propeller and jet aircraft**
- **Acute problem for general-aviation pilots graduating from single-engine aircraft**



Configurational Solutions to the Engine-Out Problem

- **Engines on the centerline (*Cessna 337 Skymaster*)**
- **More engines (*B-36*)**
- **Cross-shafting of engines (*V-22*)**
- **Large vertical tail (*Boeing 737*)**

Some Videos

XF-92A, 1948

http://www.youtube.com/watch?v=hVjaiMXvCTQ

First flight of B-58 Hustler, 1956

http://www.youtube.com/watch?v=saeejPWQTHw

Century series fighters, bombers, 1959

http://www.youtube.com/watch?v=WmseXJ7DV4c&feature=related

Bird of Prey, 1990s, and X-45, 2000s

http://www.youtube.com/watch?v=BMcuVhzCrX8&feature=related

YF-12A supersonic flight past the sun

http://www.youtube.com/watch?v=atItRcfFwgw&feature=related

Supersonic flight, sonic booms

http://www.youtube.com/watch?v=gWGLAAYdbbc&list=LP93BKTqpxbQU&index=1&feature=plcp

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Pitch-Moment Coefficient Sensitivity to Angle of Attack

For small angle of attack and no control deflection

$$\boxed{M_B = C_m \bar{q} S\bar{c} \approx \left(C_{m_o} + C_{m_\alpha}\alpha\right)\bar{q} S\bar{c}}$$