

MAE 331
AIRCRAFT FLIGHT DYNAMICS
Assignment #1
due: End-of-day, Friday, September 21, 2018



The goal of this assignment is to document the physical characteristics and flight behavior of a balsa glider. Each of you has received your own test aircraft, a *Guilow Jetfire Glider*. Balsa wood is relatively strong, but it is soft and easily broken. It absorbs water and is even softer when wet. Don't put it in your overloaded backpack; it will break. Take care when you assemble the glider and do your flight experiments. Be prepared to glue your airplane back together (SuperGlue, Duco Cement, or Elmer's Glue should work).

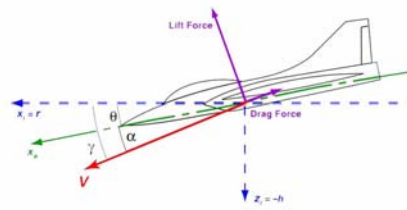
You will work in teams of two to complete the assignment and to write your report (*i.e.*, one report for each team, with the same grade assigned to each team member). The teams have been assembled with random selection. See the *Blackboard* list to identify your teammate, and start early.

Assignment

- 1) Tell everything that you know about the physical characteristics of the glider, including
 - weight
 - length

- wing span and area
- moments of inertia, center of mass
- dihedral angles, and aspect ratio of wing and tail
- wing loading
- horizontal and vertical tail height, area, and aspect ratio

- 2) From experimentation, tell everything that you know about the flight characteristics of the glider, including
- maximum range and time of flight for various launch conditions
 - average gliding speed
 - average glide path angle
 - rolling and yawing
 - effect of the glider's weight and wing position on these quantities
 - duration and radius of a loop



- 3) Use a sequence of digital photos (*e.g.*, cellphone video or photo burst) to determine altitude (h), range (r), airspeed (V), and flight path angle (γ) vs. time for a typical glide path. Plot the results.
- 4) The glide path may oscillate about the average gliding angle. Determine the period and approximate amplitude of the oscillation for one or two wing positions.
- 5) Try a few non-standard configurations, such as:
- a) Insert the wings upside down
 - b) Remove the vertical tail
 - c) Remove the horizontal tail
 - d) Tape paper ailerons to the wing to produce an “aileron roll”
 - e) Add weight to the fuselage
 - f) Cant the wingtips, either by cutting and gluing each tip, or adding cardboard “winglets”
 - g) Sweep the wings by cutting and gluing left and right wings

Does the glider fly in any of these cases? If so, how are the flight characteristics changed?

Report your flight test results with numbered sections as defined above, and add brief introduction and conclusion sections. Give each figure a number and a title, and label the

axes. Take care that the individual curves on each figure are well identified. Identify your individual contributions to the assignment.

Submit your assignment as a .pdf file via *Blackboard* (<http://blackboard.princeton.edu/>) using the name **MAE_331_HW_1**.

MAE 331
AIRCRAFT FLIGHT DYNAMICS
Assignment #2
due: End-of-day, Wednesday, October 3, 2018

The *balsa glider* shown at right was the topic of Assignment #1. In this assignment, you will compute the motions of the glider using the point-mass dynamic equations and the aerodynamic and mass characteristics that you estimate for the glider. You will work in teams of two to complete the assignment and to write your report (*i.e.*, one report for each team, with the same grade assigned to each team member). The teams have been assembled with random selection, and you have a new teammate for this assignment. See the *Blackboard* list to identify your teammate, and start early.



- 1) Estimate and plot the variation of lift and drag coefficients, C_L and C_D , as functions of the angle of attack. The angle of attack should be within a range of -2 to 16 deg. Explain how you made your lift and drag estimates. What is the maximum lift-to-drag ratio (L/D) and corresponding angle of attack?
- 2) With no thrust, the point-mass longitudinal motions of the glider are governed by four differential equations:

$$\begin{aligned}\dot{V} &= -C_D \frac{1}{2} \rho V^2 S / m - g \sin \gamma \\ \dot{\gamma} &= \frac{1}{V} \left(C_L \frac{1}{2} \rho V^2 S / m - g \cos \gamma \right) \\ \dot{h} &= V \sin \gamma \\ \dot{r} &= V \cos \gamma\end{aligned}$$

where h and r are height (altitude) and ground range. The constant airspeed V (m/s), and flight path angle γ (rad) for equilibrium glide can be found by simultaneously solving the first two equations with $\dot{V} = 0$ and $\dot{\gamma} = 0$. Plot the equilibrium-glide

values of V and γ for altitudes from 0 to 2,000 m, assuming that the L/D is maximized.

- 3) The goal is to calculate and plot flight paths for the glider using MATLAB's ode15s, ode23, or ode45. You may adapt the *Paper Airplane* code (Example 1.3-1 in *Flight Dynamics*) found at <http://www.princeton.edu/~stengel/PaperPlane.html> to solve this part of the problem. Comment on your findings, including the nature and time-scale of the response.
 - a) Using your simulation program, calculate and plot V , γ , h , and r for an equilibrium glide path during 10 sec of flight, with $L/D = (L/D)_{\max}$. Assume that the initial altitude is 10 m, and that the initial airspeed and flight path angle are the equilibrium values found in (2).
 - b) Repeat the calculation with the initial airspeed set to the equilibrium value but zero initial flight path angle.
 - c) Repeat the calculation with the equilibrium value of flight path angle but with airspeed 50% higher than equilibrium.
 - d) Beginning at the equilibrium glide altitude and flight path angle, increase the initial airspeed until the aircraft performs a loop.
 - e) Compare and plot an experimental flight path obtained in Assignment #1 with a calculated flight path that has the same initial conditions as your actual plane.
- 4) Compare and plot an experimental flight path obtained in Assignment #1 with a calculated flight path that has the same initial conditions as your actual plane.

General Guidelines

Explain what you did and discuss the significance of your findings, including comparisons with the results of Assignment #1. Arrange the sections of your flight test report with numbered sections as defined above, and add brief introduction and conclusion sections. Give each figure a number and a title, and label the axes. Take care that the individual curves on each figure are well identified. Identify your individual contributions to the assignment.

Submit your assignment as a .pdf file via *Blackboard* (<http://blackboard.princeton.edu/>) using the name **MAE_331_HW-2**.

MAE 331
AIRCRAFT FLIGHT DYNAMICS
Assignment #3
due: End of Day, October 15, 2018

The goal of this assignment is to estimate the non-dimensional "static" aerodynamic coefficients, derivatives, and control effects for the *Boeing/SAAB T-X Trainer Aircraft* [1].¹ You may use the specifications and drawings given below, what you have learned in class, the text book, and other relevant sources (see the course web page for some examples).



Figure 1. Boeing/SAAB T-X Aircraft [1].

¹ "Static" aerodynamic coefficients are those that depend on angle of attack α and sideslip angle β . "Dynamic" aerodynamic coefficients depend on body-axis angular rates, p , q , and r , and the time rates of change of α and β , i.e., $d\alpha/dt$ and $d\beta/dt$.

You will work in teams of two to complete the assignment and to write your report (i.e., one report for each team, with the same grade assigned to each team member). The teams have been assembled with random selection, and you will have new teammates for this assignment. See the *Blackboard* list to identify your teammates.

Background

The T-X is a supersonic advanced trainer that has been chosen by the USAF to replace the venerable T-38 trainer. General characteristics of the airplane are presented in [1], as deduced in [4] (Table 1). They are not official Boeing specifications, but they may be used for educational purposes. These data are consistent with the specifications for the Lockheed-Martin/KAI T-50A, Boeing/SAAB's main competitor for the program [5]. The T-50A uses the same engine and has the same design requirements as the T-X.

Table 1
General characteristics of Boeing/SAAB T-X [1]

- **Crew:** 2
- **Length:** 46.42 ft (14.15 m)
- **Wingspan:** 32.81 ft (10.00 m)
- **Aspect ratio:** 7.21 = F/A-18E/F aspect ratio
- **Wing area:** $S = b^2/AR = 27.77 \text{ m}^2$
- **Height:** 13.12 ft (4.00 m)
- **Empty weight:** 7,165 lb (3,250 kg)
- **Max takeoff weight:** 12,125 lb (5,500 kg)
- **Powerplant:** 1 × [General Electric F404](#) Afterburning turbofan, 11,000 lbf (49 kN) thrust dry, 17,700 lbf (79 kN) with afterburner
- **Maximum speed:** 808 mph (1,300 km/h; 702 kn) [$\sim M = 1.2$]
- **Range:** 1,143 mi (993 nmi; 1,839 km)
- **Service ceiling:** 50,000 ft (15,000 m)
- **Rate of climb:** 33,500 ft/min (170 m/s)



Figure 2. Frame grabs showing T-X planform (from [3]).

Photographs and company video (Fig. 1, 2, [2, 3]) clearly show that the T-X configuration is motivated as a scaled-down, single-engine version of the F/A-18E/F *Super Hornet* (Table 2). The T-X's length and wingspan are about 3/4 of the F/A-18E/F values, and thrust/weight is similar. The Northrop/McDonnell Douglas F/A-18 *Hornet* is an extension of the Northrop YF-17 *Cobra* [7]. By good fortune, NASA conducted low-speed wind tunnel tests on a scale model of the YF-17, as documented in [8]. You may refer to this memorandum in developing your aerodynamic model of the T-X aircraft.

Table 2
General characteristics of F/A-18F [6]

- **Crew:** 2
- **Length:** 60 ft 1¼ in (18.31 m)
- **Wingspan:** 44 ft 8½ in (13.62 m)
- **Height:** 16 ft (4.88 m)
- **Wing area:** 500 ft² (46.5 m²)
- **Aspect ratio:** $b^2/S = 7.21$
- **Empty weight:** 32,081 lb (14,552 kg)
- **Loaded weight:** 47,000 lb (21,320 kg)
- **Max. takeoff weight:** 66,000 lb (29,937 kg)
- **Internal fuel capacity:** 13,550 lb (6,354 kg)
- **External fuel capacity:** 5 × 480 gal tanks, totaling 16,380 lb (7,381 kg)
- **Powerplant:** 2 × General Electric F414-GE-400 turbofans
 - **Dry thrust:** 13,000 lbf (62.3 kN) each
 - **Thrust with afterburner:** 22,000 lbf (97.9 kN) each

Assignment

Because scale drawings of the T-X are not available, we will assume that its configuration is identical to the F/A-18E/F configuration shown in Fig. 3. All relative dimensions that are required for your T-X analysis should be based on this figure, defining the wingspan to be 10 m, as in Table 1, and with aspect ratio = 7.21. The grid is overlaid to make measurements of various lengths easier, but grid spacing is not scaled to

meters or feet. You can make measurements from the figure with a physical ruler or a Ruler Tool, as in *Photoshop*. Reference all measurements to the T-X wingspan given in Table 1.

- 1) Using the data tabulated above and the three-view drawing presented in Fig. 3, calculate the geometric parameters that are required to estimate the airplane's aerodynamic coefficients, including wing and tail areas and spans, airfoil chord lengths, wetted area, aspect ratios, sweep angles, taper ratios, mean aerodynamic chords, centers of pressure, dihedral angles, fuselage length and diameter, and horizontal and vertical tail lengths. Be sure to indicate what assumptions you make and what sources you use to arrive at your estimates.

- 2) Estimate and present the aircraft's non-dimensional force coefficients and derivatives assuming incompressible flow, i.e., $\mathcal{M} = 0$:
 - a) Lift slope C_{L_α} , parasitic drag C_{D_o} , and induced drag factors, ε and e , for 0° flap setting and wheels retracted.
 - b) Side-force slope C_{Y_β} .
 - c) What is the maximum lift-to-drag ratio (L/D) and corresponding angle of attack?
 - d) Using the formulas given in class, how might we expect C_{L_α} and C_{D_o} to vary for $0 < \mathcal{M} < 0.8$?

- 3) Estimate and present the aircraft's non-dimensional moment coefficients and derivatives assuming incompressible flow:
 - a) Pitch moment derivative C_{m_α} , and C_{m_o} required for flight at $(L/D)_{\max}$ for 0° flap setting and wheels retracted. Present results for three center-of-mass locations: 20%, 25%, and 30% behind the leading edge of the mean aerodynamic chord.
 - b) Roll- and yaw-moment coefficients, C_{l_β} and C_{n_β} , with C_{l_o} and C_{n_o} equal to zero, as functions of sideslip angle in $-2^\circ < \alpha < 2^\circ$. Present results for three center-of-mass locations: 20%, 25%, and 30% behind the leading edge of the mean aerodynamic chord.
 - c) Using the formulas given in class, how might we expect these values to vary for $0 < \mathcal{M} < 0.8$?

- 4) Estimate and plot the following non-dimensional control derivatives assuming incompressible flow: $C_{l_{\delta A}}$, $C_{m_{\delta E}}$, $C_{n_{\delta R}}$. Present results for three center-of-mass locations: 20%, 25%, and 30% behind the leading edge of the mean aerodynamic chord.

In addition to the material on configuration aerodynamics found in Section 2.4 of *Flight Dynamics*, the lecture slides, references included in the *Blackboard Course Materials*, and the virtual textbook (<http://www.princeton.edu/~stengel/AFDVirTex.html>), you may find web resources and the shelved and E-Library reserve books for MAE 331 helpful.

Submit your assignment via Blackboard (<http://blackboard.princeton.edu/>) using the name **MAE_331_HW_3**.

References

1. https://en.wikipedia.org/wiki/Boeing_T-X.
2. <https://www.boeing.com/defense/t-x/index.page>.
3. <https://www.boeing.com/defense/t-x/index.page#/video-player/boeing-t-x-america-s-trainer>.
4. https://www.militaryfactory.com/aircraft/detail.asp?aircraft_id=1647.
5. https://en.wikipedia.org/wiki/KAI_T-50_Golden_Eagle.
6. https://en.wikipedia.org/wiki/Boeing_F/A-18E/F_Super_Hornet.
7. https://en.wikipedia.org/wiki/Northrop_YF-17.
8. Petroff, D., Scher, S., and Sutton, C., "Low-Speed Aerodynamic Characteristics of a 0.08-Scale YF-17 Airplane Model at High Angles of Attack and Sideslip," NASA-TM-78438, Washington, Apr 1978.

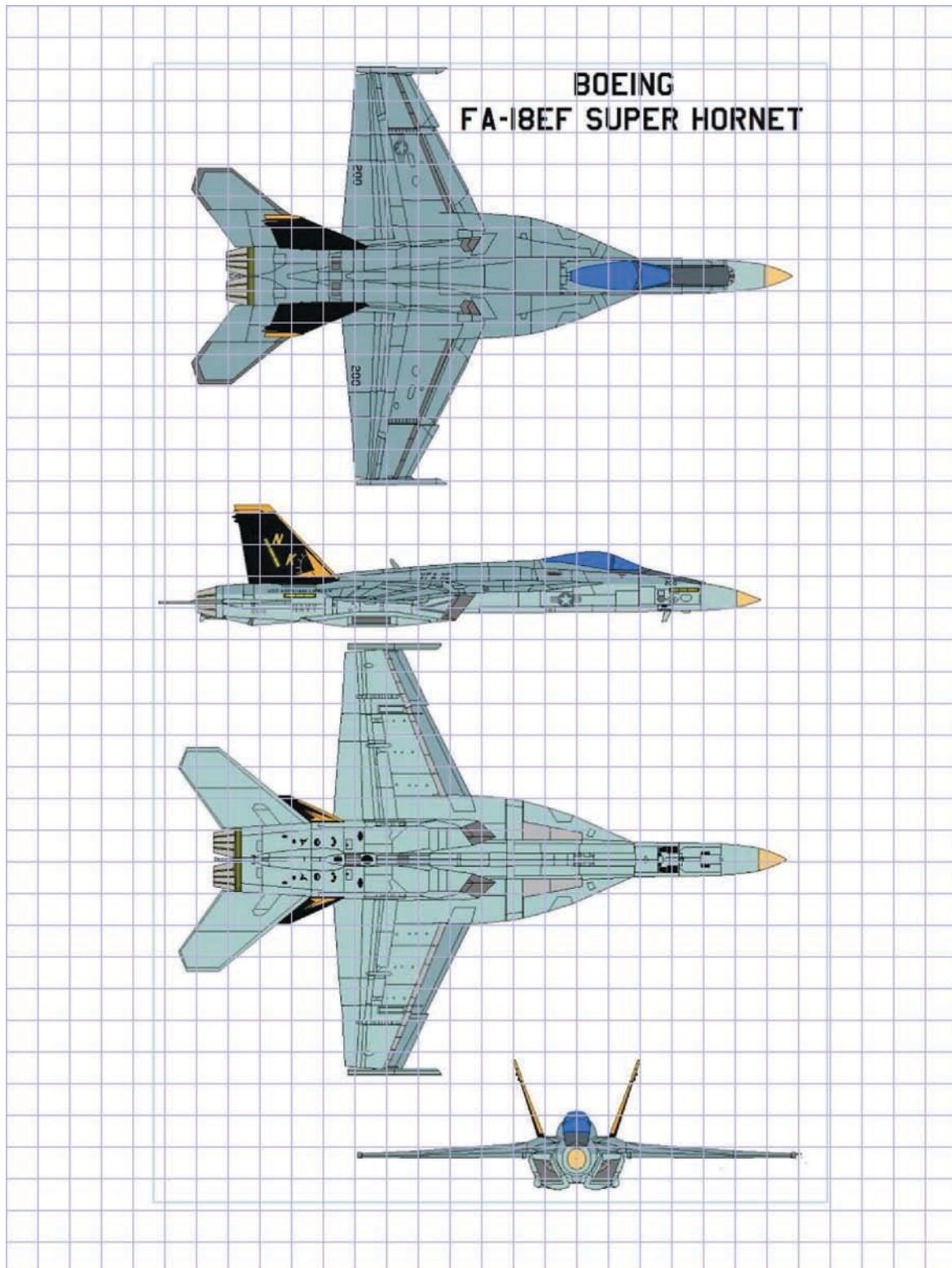


Figure 3. Three-view drawings of the *Boeing F/A-18E Aircraft*. Grid not to scale. [from <https://www.airrecognition.com/index.php?id=140>]

MAE 331
AIRCRAFT FLIGHT DYNAMICS
Assignment #4
due: End-of-Day, Friday, October 26, 2018

The goal of this assignment is to compute selected performance characteristics for the *T-X Advanced Jet Trainer* (Fig. 1). You will work in teams of two to complete the assignment and to write your report (i.e., one report for each team, with the same grade assigned to each team member). The teams have been assembled with random selection, as identified by the list on *Blackboard*.



Figure 1. *T-X Advanced Jet Trainer*.

- 1) Model the total thrust as a function of altitude from sea level to 14,000 m (see *Flight Dynamics* and Lecture 7 slides).
- 2) What are the C_L , C_D , α , and C_T , true airspeed, indicated airspeed, power, load factor, thrust/weight ratio, wing loading, and dynamic pressure during cruising flight at $(L/D)_{\max}$ and an altitude of 10,500 m? Assume the aircraft mass is 4,800 kg.
- 3) What are the achievable maximum and minimum true airspeeds for steady, level flight at an altitude of 10,500 m? Remember that there is a limit on the available thrust, and there must be enough lift to equal weight.
- 4) Calculate the maximum cruise-climb range of the *T-X Trainer* for an initial altitude of 10,000 m, an initial airplane mass of 4,800 kg, and a total fuel burn of 500 kg. Assume that the maximum sea-level thrust is 49 kN and that the maximum-thrust specific fuel consumption is 48.9 kg/kN-hr.
 - a) What are the C_L , C_D , α , and C_T during the cruise-climb?

- b) Assuming that the cruise-climb begins at an altitude of 10,000 m, what is the corresponding initial airspeed? [Solution order of (b) and (c) reversed from original spec]
 - c) What is the optimal dynamic pressure?
 - d) What is the range covered during this flight segment?
 - e) What is the final altitude?
- 5) Calculate the flight (altitude vs. true airspeed) envelope, corner velocity (equivalent airspeed), and maximum sea-level climb rate for the *T-X Trainer* at the same mass as above. Assume that the lift coefficient is linear in angle of attack up to an effective stall angle of 18° (at which point, C_L becomes constant) and that the maximum allowable load factor is 6.5 "g".

General Guidelines

Document your results in report format, e.g.,

- . Title, Course #, Assignment #, Date
- . Abstract
- . Introduction
- . Body of your work: approach, math details, graphical and numerical output
- . Conclusions
- . References

The assignments are graded on

- . Substance
- . Focus
- . Grammar and spelling; formality, sentence structure
- . Content structure and organization
- . Informative, interesting style

Explain what you did and discuss the significance of your findings. Arrange the sections of your report with sections as numbered in the assignment. Give each figure a number and a title, and label the axes. Take care that the individual curves on each figure are large enough and dark enough to convey useful information. Identify your individual contributions to the assignment.

MAE 331
AIRCRAFT FLIGHT DYNAMICS
Assignment #5
due: End of Day, Monday, November 12, 2018

The goal of this assignment is to use the MAE “Pilot Pro 4” cockpit simulator (Fig. 1, <https://www.stempilot.com/product/pilot-pro-4/>) to fly the Lockheed Martin *Prepar3d* simulation of the Cessna 172 airplane (Fig. 2). You will

- Takeoff from Princeton Airport
- Fly over the campus and Lake Carnegie
- Return to land at the Princeton Airport (Fig. 3).

You will work in teams of two to complete the assignment and will write separate reports. Each team member should complete the flight as pilot, with assistance from your teammate. Our teaching assistant, Qiang Chen, will explain how to use the simulator and give basic guidance about takeoff, navigation along your route, and landing, with technical support from Jeff Addo and Jon Prevost. Please see Qiang to arrange a meeting time and to schedule use of the simulator.



Figure 1. Aircraft cockpit simulator. Figure 2. Simulated Cessna 172 airplane.

Assignment

- 1) Fly the mission, as described above. Provide details of your flight, including
 - a. takeoff and landing airspeeds,
 - b. altitude reached
 - c. maximum bank angle
 - d. fastest and slowest speeds
 - e. screen shots of the ground track
 - f. altitude vs. range plot
 - g. time from takeoff to landing

You can record the entire flight and replay as often as possible to get additional information about your flight.

- 2) Document your work in individual reports that are organized with Abstract (one-paragraph overview), Introduction, Technical Content, Conclusion, and References. Each of you should comment on the procedures used to fly the airplane and conduct the test, as well as your qualitative evaluation of takeoff, climb, cruise, descent, and landing. Use graphics wherever appropriate. The simulator has a *Flight Analysis* graphic output that can aid in presentation.



Figure 3. Princeton Area, Google Earth.

Here are some previous results (Fig. 4):

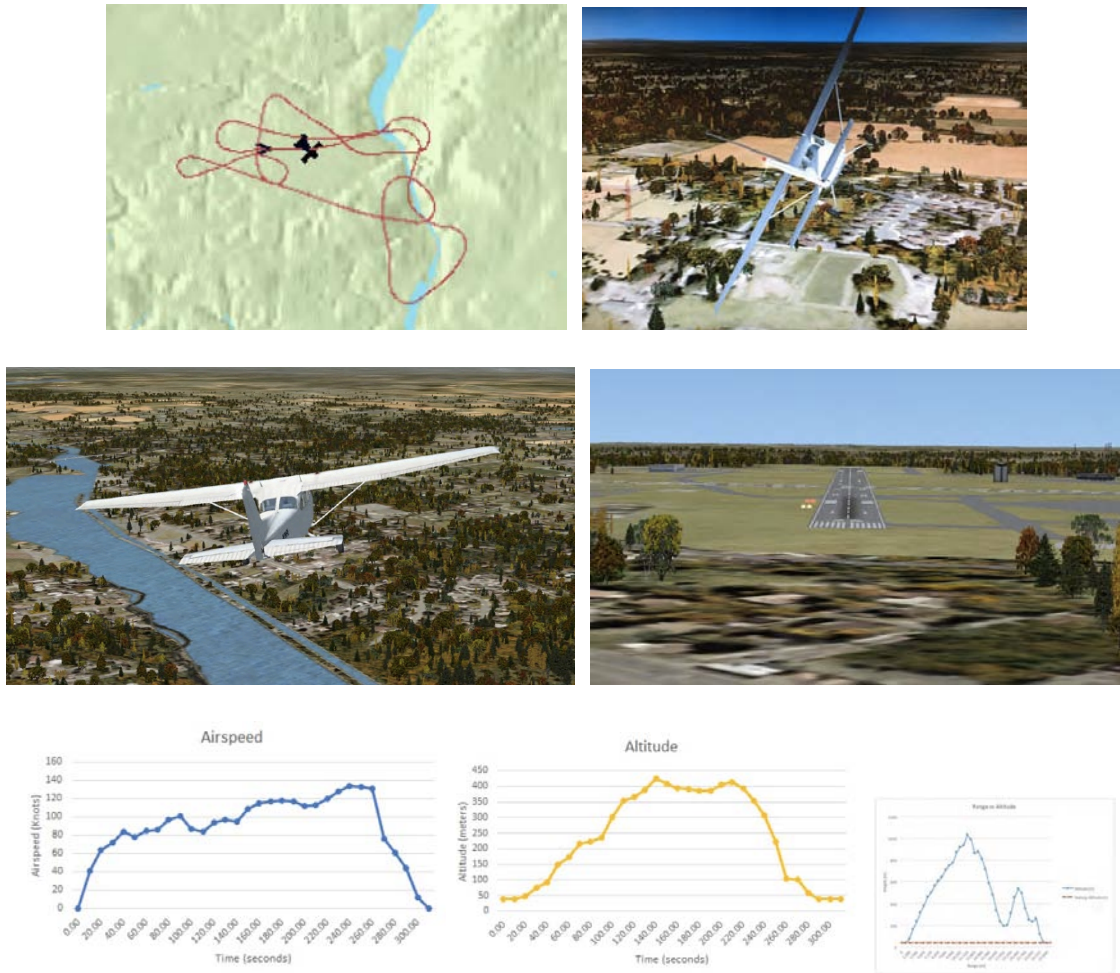


Figure 4. Prior flight simulation results.

References

1. <https://www.prepar3d.com/>
2. https://en.wikipedia.org/wiki/Microsoft_Flight_Simulator
3. https://en.wikipedia.org/wiki/Cessna_172
4. <https://disciplesofflight.com/cessna-172-skyhawk/>
5. *Edustation Flight Club Manual*, 2014 (on MAE 331 Blackboard)

MAE 331
AIRCRAFT FLIGHT DYNAMICS
Assignment #6
due: End of Day, Tuesday, November 20, 2018

This is the first of two assignments that will culminate in simulation of *T-X* (Fig. 1) maneuvers using *FLIGHT.m*, a nonlinear, six-degree-of-freedom simulation program. The goal of this assignment is to code the aerodynamic, thrust, and inertial properties of the airplane in *AeroModel.m*, based on the simplified model given below. The equations of motion are described in the MATLAB function *EoM.m* which in turn calls the function *AeroModel.m* for the inertial, aerodynamic, and thrust properties. *EoM.m* uses body-axis equations to solve for rotational and translational rates. Inertial-axis equations are used to compute translational position and the Euler angles that orient the body axes in the inertial frame.



Figure 1. T-X Advanced Jet Trainer.

FLIGHT.m and the associated functions can be found at <http://www.stengel.mycpanel.princeton.edu/FDcodeB.html>.^{*} Prior versions of *AeroModel.m* apply to two generic business jets. *AeroModelMach.m* provides inertial, aerodynamic, and thrust models for high-subsonic-Mach-number, low-angle-of-attack simulation of BizJet A, which is used for examples throughout *Flight Dynamics*.

^{*} An older version of *FLIGHT.m* is located at <http://www.princeton.edu/~stengel/FDcodeB.html>. The university is in the process of transferring to a new web hosting system, *cPanel.com*. All of my web pages are on both systems, but I am updating pages only on the new system. The old version also could be used for the assignment; however, there are minor I/O differences.

AeroModelAlpha.m uses aerodynamic data that is suitable for low-Mach-number, high-angle-of-attack simulation of a similar aircraft, BizJet B. This assignment and the next focus on the Mach-dependent characteristics of the *T-X* trainer at low angles of attack and subsonic Mach number (although the actual trainer will be capable of flight to Mach 1.2). The goal of this assignment is to create a new function, *AeroModelUser.m*, by modifying *AeroModelMach.m* to model the *T-X* with the constants given below. You will work in teams of two, as identified on *Blackboard*.

The state and control vectors employed in *FLIGHT.m* are

$$\mathbf{x}(t) = \begin{bmatrix} u(t) & v(t) & w(t) & x(t) & y(t) & z(t) & p(t) & q(t) & r(t) & \phi(t) & \theta(t) & \psi(t) \end{bmatrix}^T \quad (1)$$

$$\mathbf{u}(t) = \begin{bmatrix} \delta E(t) & \delta A(t) & \delta R(t) & \delta T(t) & \delta AS(t) & \delta F(t) & \delta S(t) \end{bmatrix}^T \quad (2)$$

Inertial properties and dimensions for the aircraft are given in Table 1; these properties are fixed for the assignment. The maximum thrust at any altitude, h , is given by eq. 3; it is throttled by $u(4)$.

FLIGHT.m trims the airplane flight condition using the stabilator, δS . The *T-X* has an all-moving horizontal tail, and so the elevator and the stabilator are the same. Assume that the stabilator $[u(7)]$ effect is identical to the elevator $[u(1)]$ effect. This assures that the trimming routine $[fminsearch$ of *TrimCost.m*] works properly. Landing gear, flap setting $[u(6)]$, and asymmetric spoiler $[u(5)]$ effects are neglected.

Incompressible-flow aerodynamic coefficients for the *T-X* are presented in Tables 2 and 3. To simulate the effects of compressibility, the incompressible coefficients $[C_D, C_L, C_m, C_Y, C_l, C_n]_{inc}$ each should be multiplied by the Mach-dependent factor, $f(\mathcal{M})$, (Table 4) in your version of *AeroModel.m*.

Assignment

To check your code, assume that $\mathcal{M} = 0.75$ and $h = 10,500$ m.

- 1) Revise the mass and inertial properties in *AeroModelMach.m*.
- 2) Revise the thrust computation in *AeroModelMach.m*.
- 3) Revise the calculations for $[C_D, C_L, C_m, C_Y, C_l, C_n]_{comp}$ in *AeroModelMach.m*.
- 4) Store the resulting model as *AeroModelUser.m*.
- 5) Assure that the inputs and outputs for *AeroModelUser.m* are compatible with *FLIGHT.m* so that simulations can be conducted in the next assignment. This is done by opening *FLIGHT.m*, setting `MODEL = 2`, and running that script. Set `TRIM = 1` and `LINEAR = 1`, allowing the trim condition and the linear model matrices at the trim point, \mathbf{F} and \mathbf{G} , to be calculated.

AERODYNAMIC AND INERTIAL MODEL OF THE T-X TRAINER

Table 1. Inertial Properties and Dimensions

m	4800 kg
I_{xx}	20950 kg-m ²
I_{yy}	49675 kg-m ²
I_{zz}	62525 kg-m ²
I_{xz}	-1710 kg-m ²
\bar{c}	3.03 m
b	10 m
S	27.77 m ²

Thrust Model

$$T_{max} = T_{SL} \left[\frac{\rho(h)}{\rho_{SL}} \right] \left[1 - e^{\left(\frac{h-15000}{2000} \right)} \right] \quad (3)$$

$$T_{SL} = 49,000 \text{ N}$$

Table 2. Drag, Lift, and Pitching Moment Coefficients and Derivatives (Incompressible Flow)

C_{D_o}	0.019	C_{L_o}	0	C_{m_o}	0
ϵ	0.093	C_{L_α}	4.92	SM	0.2
		$C_{L_{\dot{q}}}$	2.49	$C_{m_{\dot{q}}}$	-4.3
		$C_{L_{\delta E}}$	0.72	$C_{m_{\delta E}}$	-1.25
		$C_{L_{\delta S}}$	0.72	$C_{m_{\delta S}}$	-1.25

Table 3. Side Force, Rolling Moment, and Yawing Moment Coefficients and Derivatives (Incompressible Flow)

C_{Y_o}	0	C_{l_o}	0	C_{n_o}	0
C_{Y_β}	-0.5	C_{l_β}	-0.066	C_{n_β}	0.37
$C_{Y_{\dot{\beta}}}$	0	$C_{l_{\dot{\beta}}}$	-0.5	$C_{n_{\dot{\beta}}}$	-0.06
$C_{Y_{\ddot{\beta}}}$	0	$C_{l_{\ddot{\beta}}}$	-0.05	$C_{n_{\ddot{\beta}}}$	-0.5
$C_{Y_{\delta A}}$	0	$C_{l_{\delta A}}$	0.12	$C_{n_{\delta A}}$	0
$C_{Y_{\delta R}}$	0.04	$C_{l_{\delta R}}$	0.03	$C_{n_{\delta R}}$	-0.2

Table 4. Compressibility Effect (Prandtl Factor), $f(\mathcal{M})$, vs. Mach Number

Mach	Multiplier, $f(\mathcal{M})$	Mach	Multiplier, $f(\mathcal{M})$
0	1.0000	0.74	1.0147
0.10	1.0000	0.75	1.0185
0.20	1.0000	0.76	1.0349
0.30	1.0000	0.77	1.0512
0.40	1.0000	0.78	1.0800
0.50	1.0000	0.79	1.1185
0.60	1.0000	0.80	1.1810
0.70	1.0031	0.81	1.2675
0.71	1.0060	0.82	1.3637
0.72	1.0089	0.83	1.4791
0.73	1.0118	0.84	1.6041

MAE 331
AIRCRAFT FLIGHT DYNAMICS
Assignment #7
due: End of Day, Wednesday, December 5, 2018

This is the second of two assignments that culminate in simulation of T - X maneuvers using *FLIGHT.m*, a nonlinear, six-degree-of-freedom simulation program. The goal of this assignment is to assess the aircraft's response at various flight conditions using aerodynamic, thrust, and inertial properties of the airplane specified in Assignment 6. These properties are coded in *AeroModelUser.m*, which can be found at <http://www.stengel.mycpanel.princeton.edu/FDcodeB.html>. You will work in teams of two, as identified on *Blackboard*.



Figure 1. T - X Advanced Jet Trainer.

We want to simulate the T - X 's trim settings, response to initial conditions and control inputs, steady turning, and ability to perform a few maneuvers.

Assignment

- 1) Calculate the trimmed values of throttle setting δT , stabilator δS , and pitch angle θ , for $\mathcal{M}=0.75$ and $h=1,000, 5,000, 10,000,$ and $14,000$ m. Give the corresponding true and indicated airspeeds at these points.

- 2) Starting from a trimmed flight condition with $\mathcal{M}=0.75$ and $h=10,500$ m, compute, plot, and comment on response to the following initial conditions:
- $\Delta q(0) = 1^\circ/\text{s}$. Compute state history for $0 < t < 20$ s.
 - $\Delta r(0) = 1^\circ/\text{s}$. Compute state history for $0 < t < 20$ s.
 - $\Delta u(0) = 1$ m/s. Compute state history for $0 < t < 100$ s.

What can you deduce about the longitudinal and lateral-directional modes of motion?

- 3) Starting from a trimmed flight condition with $\mathcal{M}=0.75$ and $h=10,500$ m, compute, plot, and comment on response to the following initial conditions:
- Step Input*: $\Delta \delta E(t) = 1^\circ$; hold for 60 s. Compute state history for $0 < t < 60$ s.
 - Step Input*: $\Delta \delta T(t) = 1\%$ of trimmed value; hold for 60 s. Compute state history for $0 < t < 60$ s.
 - Step Input*: $\Delta \delta R(t) = 1^\circ$; hold for 60 s. Compute state history for $0 < t < 60$ s.
 - Impulse Input*: $\Delta \delta A(t) = 1^\circ, t = 0$ to 0.5 s; $= 0^\circ, t = 0.5$ to 60 s.

What can you deduce about the effects of control on longitudinal and lateral-directional modes of motion?

- 4) Starting from a trimmed flight condition with $\mathcal{M}=0.75$ and $h=10,500$ m, where the roll angle is zero, specify *tuHis* and *deluHis* to produce a steady, coordinated banked turn ($\beta < 1^\circ$) at fixed airspeed and altitude (see “Control Perturbation History,” around line 200 in the code for example of a control history). The roll angle, ϕ , should be held constant at approximately $10^\circ (\pm 1^\circ)$ during the interval from $t = 10$ to 30 sec. If natural damping of the Dutch roll mode is too light for this flight condition, assume that an electronic yaw damper increases C_{n_r} by a factor of 8 to 10. You may use a “cut-and-try” approach to finding the control history.¹ Plot angular velocities, Euler angles, altitude vs. time, North vs. East position, and control histories.

¹ For example, First specify time at 5-sec time intervals, *tuhis*, for the control from 0 to 50 s. Then, find aileron values, *deluhis*(2), at each time that put the roll angle in the specified range. The sideslip angle is too high; adjust the rudder, *deluhis* (3), to bring the sideslip angle below 1° . Too much altitude is lost; adjust the elevator, *deluhis* (1), to increase angle of attack and reduce altitude loss. The increased drag decreases speed; increase the throttle setting, *deluhis* (4), to reduce the loss.

MAE 331
AIRCRAFT FLIGHT DYNAMICS
Assignment #8
due: 5 pm, December 14, 2018



Figure 1. *T-X Advanced Jet Trainer.*

The goal of this assignment is to analyze linear dynamic models of the *T-X trainer* (Fig. 1). You begin by using *FLIGHT.m* with *AeroModelUser.m* to compute trimmed (nominal) flight conditions and the corresponding (12 x 12) **F** and (12 x 7) **G** matrices of the linear, time-invariant (LTI) system model. The matrices are saved to *Fmodel* and *Gmodel* in *FmodelFile.mat* and *GmodelFile.mat*. You can use these matrices in your own code, or you can use *SURVEY.m*, which can be found at <http://www.stengel.mycpanel.princeton.edu/FDcodeC.html>. *SURVEY.m* makes calculations that you choose by setting “flags” at the beginning of the script. Analyses can be based on the original 12th-order LTI model, a 12th-order model reordered in longitudinal and lateral-directional parts, or various reduced-order longitudinal and lateral-directional models. For either a longitudinal or lateral-directional option and a nominal flight condition, *SURVEY.m* computes:

- Transient time response to initial conditions and control inputs
- Eigenvalues and eigenvectors
- Natural frequencies, damping ratios, and time constants
- Equilibrium response to constant controls and commands
- Controllability and observability
- Transfer functions with factored numerators and denominators
- Frequency response (Bode plots, Nyquist plots, and Nichols charts)
- Pilot-Aircraft Interaction
- Root locus analysis of proportional feedback control

Two nominal flight conditions are to be investigated:

- A. Typical cruising condition: $\mathcal{M} = 0.75, h = 10,500 \text{ m}$
 B. Pre-landing approach: $IAS = 140 \text{ kt}, h = 500 \text{ m}$

You will analyze 4th-order hybrid longitudinal models with state and control vectors,

$$\Delta \mathbf{x}(t) = \begin{bmatrix} \Delta V(t) & \Delta \gamma(t) & \Delta q(t) & \Delta \alpha(t) \end{bmatrix}^T, \quad \Delta \mathbf{u}(t) = \begin{bmatrix} \Delta \delta E(t) & \Delta \delta T(t) \end{bmatrix}^T$$

and 4th-order stability-axis lateral directional models with state and control vectors,

$$\Delta \mathbf{x}(t) = \begin{bmatrix} \Delta r(t) & \Delta \beta(t) & \Delta p(t) & \Delta \phi(t) \end{bmatrix}^T, \quad \Delta \mathbf{u}(t) = \begin{bmatrix} \Delta \delta A(t) & \Delta \delta R(t) \end{bmatrix}^T$$

Assignment

- 1) Run *FLIGHT.m* to generate and save the 12th-order linear models for Cases A and B.
- 2) If you use *SURVEY.m*, set the LLD, SBA, LON, and DIM flags to generate 4th-order longitudinal and lateral-directional hybrid/stability-axis models. Compute and comment on the eigenvalues, natural frequencies, periods, damping ratios, and time constants of \mathbf{F}_{Lon} and \mathbf{F}_{LD} at the two nominal flight conditions.
- 3) Plot the first 40 sec of longitudinal response to unit initial conditions (m/s and deg/s) in ΔV and Δq for both cases. How do the results compare with each other and with results of HW #7?
- 4) Plot the first 40 sec of lateral-directional control response for both cases:
 - a) *Step Input*: $\Delta \delta R(t) = 1^\circ$; hold for 60 s. Compute state history for $0 < t < 60 \text{ s}$.
 - b) *Impulse Input*: $\Delta \delta A(t) = 1^\circ, t = 0 \text{ to } 0.5 \text{ s}; = 0^\circ, t = 0.5 \text{ to } 60 \text{ s}$.
 How do the results compare with each other and with results of HW #7?
- 5) Compute and describe the following transfer functions for Case A:
 - a) $\Delta \alpha(s)/\Delta \delta E(s), \Delta q(s)/\Delta \delta E(s), \Delta \theta(s)/\Delta \delta E(s)$
 - b) $\Delta V(s)/\Delta \delta T(s), \Delta \gamma(s)/\Delta \delta T(s)$
 - c) $\Delta p(s)/\Delta \delta A(s), \Delta r(s)/\Delta \delta A(s), \Delta \beta(s)/\Delta \delta A(s)$
 - d) $\Delta p(s)/\Delta \delta R(s), \Delta r(s)/\Delta \delta R(s), \Delta \beta(s)/\Delta \delta R(s)$
- 6) Present the Bode plots corresponding to the $\Delta \alpha(s)/\Delta \delta E(s), \Delta V(s)/\Delta \delta T(s), \Delta \beta(s)/\Delta \delta R(s)$, and $\Delta p(s)/\Delta \delta A(s)$ transfer functions for Case A. Using *asympt.m*, indicate the zero and pole frequencies on each plot, as well as the slopes of the amplitude-ratio asymptotes (dB/dec) between the singularities.
- 7) A simple model for pilot control dynamics is $\mathcal{H}(s)_{pilot} = K_p \left[\frac{1/T_p}{s + 1/T_p} \right]$, with the time constant, $T_p = 0.25 \text{ s}$. We want to evaluate the effect of pilot dynamics on

feedback control for longitudinal and lateral-directional cruising flight at Case *A*. This can be done either by modifying *SURVEY.m* or by exporting transfer functions (Part 5) to your own script. Plot the root locus for positive and negative root locus gain K_p in the following cases:

a) $\mathcal{H}(s)_{aircraft} = K_p \Delta n_z(s) / \Delta \delta E(s)$

b) $\mathcal{H}(s)_{pilot-aircraft} = \mathcal{H}(s)_{pilot} \Delta n_z(s) / \Delta \delta E(s)$

c) $\mathcal{H}(s)_{aircraft} = K_p \Delta p(s) / \Delta \delta A(s)$

d) $\mathcal{H}(s)_{pilot-aircraft} = \mathcal{H}(s)_{pilot} \Delta p(s) / \Delta \delta A(s)$

- 8) Just one more thing: How important is the effect of air density variation on the airplane's modes of motion? For a quick answer, recalculate the Case A longitudinal eigenvalues, natural frequencies, periods, damping ratios, and time constants with DIM=6, and compare them to your previous results (Part 2).

Document your work in report format, organized with Abstract (one-paragraph overview), Introduction, Technical Content, Conclusion, and References.